Appendix A

The application of Goda's breaking method in spreadsheet

Goda's breaking method requires offshore wave conditions [23]. In the event that the given wave height is not offshore, a synthetic one is produced, as explained below in the following:
The local wave height at a given water depth is given. The deepwater wavelength is calculated (Eq.A.1), and the breaker limit wave height is estimated using Goda's breaking criterion (Eq. A.2).

$$\mathbf{L_0 = \frac{gT_p{}^2}{2\pi}} \qquad (A.1)$$

Where $\quad L_0$*= offshore wavelength*
T_p*= peak period*

$$\mathbf{\frac{H_b}{L_0} = A\left\{1 - \exp\left[-1.5\frac{\pi d_b}{L_o}\left(1 + 15\left(\frac{1}{m}\right)^{\frac{4}{3}}\right)\right]\right\}} \qquad (A.2)$$

Where $\quad H_b$= breaking wave height
A = coefficient set equal to 0.12
m = bed slope (1:)

The given wave height is compared with the limiting wave height, and a warning is given if this has been exceeded; in this case, there is no need to proceed with the method.
If the initial wave height is smaller than the limiting wave height, the local wavelength is determined, using either Fenton's formula (Eq.A.3), for intermediate water, or the formula for shallow water (Eq.A.4).

$$\mathbf{L_{local} = L_0 \tanh\frac{2}{3}\left(\frac{2\pi h_{local}}{L_0}\right)^{\frac{3}{4}}} \qquad \mathbf{0.04 \le \frac{h_{local}}{L_{local}} \le 0.5} \qquad (A.3)$$

$$\mathbf{L_{local} = T\sqrt{gh_{local}}} \qquad \mathbf{\frac{h_{local}}{L_{local}} < 0.04} \qquad (A.4)$$

Where L_{local}= wavelength calculated at a given water depth
h_{local}= initial water depth

The shoaling coefficient K_s is then estimated. Since non-linear effects can be neglected in relative deep water [20], the shoaling coefficient is calculated here using the small amplitude wave theory (Eq.A.5).

$$\mathbf{K_s = \frac{1}{\sqrt{\left[1 + \frac{\left(\frac{4\pi h_{local}}{L_{local}}\right)}{\sinh\left(\frac{4\pi h_{local}}{L_{local}}\right)}\right]\tanh\left(\frac{2\pi h_{local}}{L_{local}}\right)}}} \qquad (A.5)$$

From the relationship relating the offshore wave height to the local wave height (Eq.A.6) a synthetic offshore wave height is derived.

$$\mathbf{H_{s0} = \frac{H_{slocal}}{K_s}} \qquad (A.6)$$

The equivalent significant deepwater wave height (significant deepwater wave height after being refracted) is calculated (Eq.A.7).

$$\mathbf{H_{s0}{}' = K_r H_{s0}} \qquad (A.7)$$

Where K_r= refraction coefficient

Coming inshore, the shoaling coefficient (Shuto's non-linear shoaling coefficient, as suggested in Goda [20] is then estimated (Eq.A.8) and the wave height is determined Eq. (A.9).

$$K_s = K_{si} \qquad \text{for } h_{30} \le h$$
$$\mathbf{K_s = (K_{si})_{30}\left(\frac{h_{30}}{h}\right)^{\frac{2}{7}} \qquad for\ h_{50} \le h \le h_{30}} \tag{A.8}$$
$$K_s\left(\sqrt{K_s} - B\right) - C = 0 \qquad \text{for } h < h_{50}$$

where h= water depth

K_{si} = shoaling coefficient for small amplitude wave Eq. (A.5)

h_{30} = water depth satisfying Eq. (A.9)

$(K_{si})_{30}$= shoaling coefficient for h_{30}

h_{50} = water depth satisfying Eq. (A.10)

B, C = constants defined in Eq. (A.11) and Eq. (A.12)

$$\mathbf{\left(\frac{h_{30}}{L_0}\right)^2 = \frac{2\pi}{30}\frac{H_{s0}'}{L_0}(K_{si})_{30}} \tag{A.9}$$
$$\mathbf{\left(\frac{h_{50}}{L_0}\right)^2 = \frac{2\pi}{50}\frac{H_{s0}'}{L_0}(K_{si})_{50}} \tag{A.10}$$
$$\mathbf{B = \frac{2\sqrt{3}}{\sqrt{\frac{2\pi H_{s0}'}{L_0}}}\frac{h}{L_0}} \tag{A.11}$$
$$\mathbf{C = \frac{C_{50}}{\sqrt{\frac{2\pi H_{s0}'}{L_0}}}\left(\frac{L_0}{h}\right)^{\frac{3}{2}}} \tag{A.12}$$

where $(K_{si})_{30}$= shoaling coefficient at $h=h_{50}$

C_{50} = constant defined by Eq. (A.13)

$$\mathbf{C_{50} = (K_{si})_{50}\left(\frac{h_{50}}{L_0}\right)^{\frac{3}{2}}\left[\sqrt{2\pi\frac{H_{s0}'}{L_0}(K_{si})_{50}} - 2\sqrt{3}\frac{h_{50}}{L_0}\right]} \tag{A.13}$$

The wave height is then estimated by shoaling, Eq. (A.14) and compared with the breaker limit wave height, calculated using Goda's breaking criterion (Eq. A.2).

$$\mathbf{H_{si} = K_s H_{s0}'} \tag{A.14}$$

When the limit is exceeded, breaking is initiated, the wave has entered the surf zone and Goda's braking method is applied (Eq. A.15).

$$\mathbf{H_{1/3} = \begin{cases} K_s H_{s0}' & for\ \frac{h}{L_0} \ge 0.2 \\ \min\{(\beta_0 H_{s0}' + \beta_1 h), \beta_{max} H_{s0}', K_s H_{s0}'\}\ for & \frac{h}{L_0} < 0.2 \end{cases}} \tag{A.15}$$

where β_0, β_1, and β_{max} are defined as follow:

$$\beta_0 = 0.028\left(\frac{H_{s0}'}{L_0}\right)^{-0.38}\exp(20m^{1.5}) \tag{A.16}$$

$$\beta_1 = 0.52e^{4.2m} \tag{A.17}$$
$$\beta_{max} = \max\left\{0.92, 0.32\left(\frac{H_{s0}'}{L_0}\right)^{-0.29} e^{2.4m}\right\} \tag{A.18}$$

Appendix B

Formulae:

a) Miche [27] developed the semi-theoretical breaking criterion for periodic waves in finite water depth and express the limiting steepness for progressive waves in any depth of water as

$$\frac{H_b}{L_b} = 0.142\tanh\left(\frac{2\pi d_b}{L_b}\right) \tag{B.1}$$

where L_b is the wavelength at breaking (i.e., immediately seaward of the breaker). This formula is only strictly valid only for a horizontal bottom.

b) Collins [26] produced a breaking height formula from linear wave theory and empirically included the slope effect into the formula. His equation was expressed as

$$H_b = d_b(0.72 + 5.6m) \tag{B.2}$$

c) Weggel [31] proposed an empirical formula for computing breaking wave height. The formula is valid for a range of 1/50 < m <1/5. The formula was expressed as

$$H_b = \frac{d_b g T^2 1.56/[1+e^{-19.5m}]}{gT^2 + d_b 43.75[1-e^{-19m}]} \tag{B.3}$$

d) Sakai and Battjes [28], based on the theory of Cokelet [34], plotted a curve of the wave breaking limit as a function of H_b/H_0 against H_0/L_0. This curve can be described by the following equations

$$H_b = H_0[0.3839(H_0/L_0)^{-0.3118}] \text{ when } \frac{H_0}{L_0} < 0.0208 \tag{B.4a}$$

$$H_b = H_0\left[0.6683\left(\frac{H_0}{L_0}\right)^{-0.1686}\right] \quad \text{when } 0.0208 \le \frac{H_0}{L_0} < 0.1 \tag{B.4b}$$

$$H_b = H_0 \qquad \text{when } 0.1 \le \frac{H_0}{L_0} \tag{B.4c}$$

The same curve in Sakai and Battjes (1980) also represents the ration of H_b/H_0 against d_b/L_0, which can be recast into the equations

$$H_b = H_0\left[27429\left(\frac{d_b}{L_0}\right)^2 - 773.71\left(\frac{d_b}{L_0}\right) + 7.4343\right] \text{ when } \quad \frac{d_b}{L_0} < 0.011 \tag{B.5a}$$

$$H_b = H_0\left[0.3976\left(\frac{d_b}{L_0}\right)^{-0.3834}\right] \quad \text{when } 0.011 \le \frac{d_b}{L_0} < 0.049 \tag{B.5b}$$

$$H_b = H_0\left[21.867\left(\frac{d_b}{L_0}\right)^2 - 7.06\left(\frac{d_b}{L_0}\right) + 1.5573\right] \quad \text{when } 0.049 \le \frac{d_b}{L_0} < 0.6 \tag{B.5c}$$

$$H_b = H_0 \qquad \text{when } 0.6 \le \frac{d_b}{L_0} \tag{B.5d}$$

e) Fenton and McKee (1990) determined the greatest unbroken wave that could exist as a function of both wavelength and depth over a nearly horizontal bottom as

f)

$$H_b = d_b \frac{[0.141063\,(L_b/d_b) + 0.0095721\,(L_b/d_b)^2 + 0.0077829\,(L_b/d_b)^3]}{[1 + 0.078834\,(L_b/d_b) + 0.0317567\,(L_b/d_b)^2 + 0.0093407\,(L_b/d_b)^3]} \tag{B.6}$$

g) Kaminsky and Kraus (1993) ,based on the analysis of large data set on depth-limited breaking of regular waves incident to plane sloping beaches, derived a breaking height and a breaking depth formulae

$$H_b = 0.46H_0\left(\frac{H_0}{L_0}\right)^{-0.28} \quad \text{(B.7)}$$

$$d_b = 0.3m^{-0.25}H_0\left(\frac{H_0}{L_0}\right)^{-0.23} \quad \text{(B.8)}$$

h) Komar (1998) proposed two separate equations for H_b and d_b, respectively.

$$H_b = 0.39g^{0.2}\left(TH_0{}^2\right)^{0.4} \quad \text{(B.9)}$$

$$d_b = H_b\{1.2[m/(H_b/L_b)^{0.5}]^{0.27}\} \quad \text{(B.10)}$$

i) Rattanapitikon and Shibayama (2006) developed a breaking depth and wave height formulae based on the reanalysis of existing formulas which gave good predictions for small- and large-scale experiments.

j)

$$\mathbf{h_b = (3.86m^2 - 1.98m + 0.88)H_0\left(\frac{H_0}{L_0}\right)^{-0.16} \; for \; \frac{H_0}{L_0} \leq 0.1} \quad \textit{(B.11a)}$$

$$h_b = (3.86m^2 - 1.98m + 0.88)H_0\left(\frac{H_0}{L_0}\right)^{-0.34} \quad \text{for } \frac{H_0}{L_0} > 0.1 \quad \text{(B.11b)}$$

and

$$\mathbf{H_b = (-0.57m^2 + 0.31m + 0.58)L_0\left(\frac{H_0}{L_0}\right)^{0.83}} \quad \textit{(B.11c)}$$

where m is the bed slope.

k) Le Roux (2007) , proposed two separate equations for fully developed waves, H_b and d_b, respectively

$$H_b = L_0/24 \quad \text{(B.12)}$$

$$d_b = L_0/20.0392 \quad \text{(B.13)}$$