In-Plane Membrane Ordering and Topological Defects

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Abstract

Intrinsic properties of biological membrane constituents dominantly influence the shape of biological membranes and consequently their functionality. This impact is in particular pronounced if anin-plane membrane order is present. In such cases topological defects (TDs) inevitable appear if membranes' topology is different from the toroidal one. TDs give rise to spatial non homogeneities which might trigger various functional responses in membranes, such as membrane fission, tabulation and vesiculation. In the paper we describe minimal continuum-type models which consider in-plane ordering within membranes focusing on TDs. We consider cases with nematic-type orientational (quadrupolar) ordering. We first present the Deviatoric Curvature (DC) model and the Minimal Extrinsic Curvature (MEC) model, respectively. The DC model demonstrates that the deviatoric curvature plays similar role as an external ordering field. Using the MEC model we show that membrane structures possess TDs which obey classical topological theorems. Finally, we present the Landau-type modelling which is particularly adequate in studies focusing on impacts of TDs.

Keywords: Biological Membranes; Nematic Ordering; Topological Defects

Introduction

Membranes of biological cells are complex systems consisting of lipid molecules, proteins and many other components [1]. Their key properties could be inferred using a relatively simple continuum-type geometrically based models. Within them membrane configurations are determined using a two dimensional (2D) curvature field [1-3]. The diversity of different membrane configurations enormously increases if a global or local membrane in-plane ordering is present [4-8]. This could arise due to anisotropic shapes of membrane embedded constituents [3,4,6,9], tilted lipid acyl chains within membrane layers [7], membrane attached anisotropic proteins [8], or by some other means. Resulting ordering is commonly modelled by the so-called p-atic fields [7], invariant under in-plane rotations for $2\pi / p$. Most cases refer to p=1, p=2 and p=6, where ordering fields could be described by a vector field, quadrupolar (nematic) field and complex hexatic order parameter field, respectively.

If an in-plane ordering exists it could exhibit topological defects (TDs) [11,12]. TDs are the consequence of symmetry breaking mechanisms which

are ubiquitous in nature. They could be locally stabilized by topological factors where microscopic details are not relevant. Consequently, physics of topological defects exhibits several universalities [11-13], spanning the physics of condensed matter systems, particle physics and cosmology. Deep understanding of TDs might unravel fundamental workings of nature. Namely, there are strong evidences that *fields* represent fundamental entities of nature [14]. If this is the case then the *fundamental particles* are emergent phenomena and TDs might explain their local stability, as originally suggested by Lord Kelvin and mathematically demonstrated by Skyrme [15].

In our study we will consider point TDs in effectively 2D systems. Some representative TDs for *p*=2 type fields exhibiting the head-to-tail invariance are shown in Figure 1. At origin of TDs the relevant field is not uniquely defined and is consequently melted. The linear size of the defect core, where ordering is essentially melted, is given by the relevant order parameter correlation length ξ. The key property of TDs in 2D is their winding number m [11,12]. It determines a number of rotations of the field along a line enclosing the defect core counter clock-wise. TDs with $m>0$ and $m<0$ are commonly referred as *defects* and *antidefects*, respectively. The total charge of TDs within a system is conserved for fixed boundary conditions (if the field is not melted). In several cases TDs behave similar to electric charges, i.e., a pair of like charges repels each other and TDs of opposite charges attract each other. Therefore, pairs {+*m*,-*m*} tend to annihilate into a defect-less state [16].
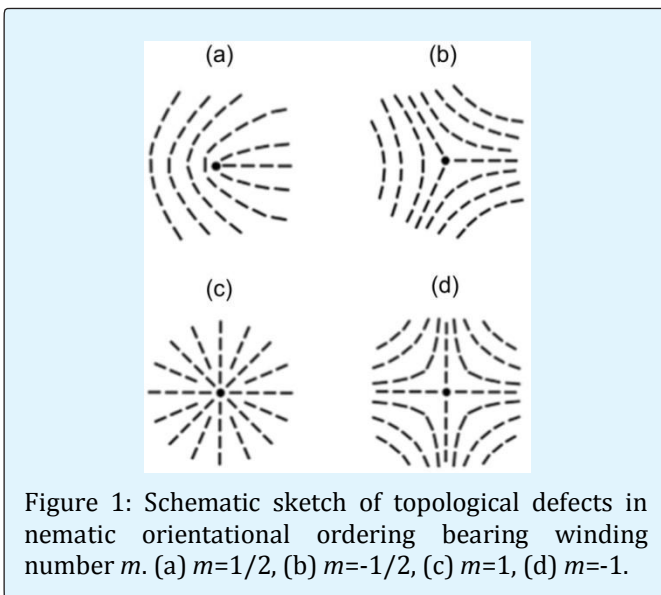

Figure 1: Schematic sketch of topological defects in nematic orientational ordering bearing winding number *m*. (a) *m*=1/2, (b) *m*=-1/2, (c) *m*=1, (d) *m*=-1.

In biological membranes exhibiting in-plane ordering TDs unavoidable form for topologies different from the toroidal one [17]. They give rise to localized non-homogeneities which might be involved, or even trigger various cellular features (e.g., budding, cell fission, cross-membrane transport) [1,5,9]. In this paper we consider common models of membranes used to study their in-plane ordering.

Stability of Topological Defects

The total winding number m_{tot} hosted by a closed membrane exhibiting in-plane ordering is given by Gauss-Bonnet and Poincare-Hopf theorems [18]:

$$m_{tot} = \chi = 2(1-g) = \frac{1}{2\pi} \oiint_{\zeta} K d^2 \vec{r} .$$

Here χ stands for the Euler characteristics of the surface ζ, g is the genus (number of holes within the surface), and K is the Gauss curvature of an infinitesimally small surface patch $d\zeta = d^2\vec{r}$. For example, for spherical (toroidal) structure it holds m_{tot}=2 (m_{tot}=0). Number of TDs for a given geometry is *p*-atic *field* dependent. In planar geometries the energy cost ΔF of a topological defect scales as $\Delta F \propto m^2$ [19,20]. Consequently, if one enforces within a region a topological defect of charge m it tends to decay into *pelementary* T Dsbearing charges m_0=1/*p*. Therefore, for a simple spherical geometry one expects four *m*=1/2 TDs.

Furthermore, spatially nonhomogeneous Gauss curvature could have strong impact on the number and positions of TDs. Interaction between K and TDs is well embodied in the Effective Topological Charge Cancellation (ETCC) mechanism [21]. In it one assigns an effective topological charge $\Delta m_{eff} = \Delta m + \Delta m_K$ to each surface patch $\Delta\zeta$ characterized by its average characteristic Gaussian curvature

$$\overline{K} = \frac{1}{\Delta\zeta} \iint_{\Delta\zeta} K d^2\vec{r}. \qquad (2)$$

The quantity Δm refers to the topological charge of "real" TDs. On the other hand Δm_K represents the spread curvature topological charge defined by

$$\Delta m_K = -\frac{1}{2\pi} \iint_{\Delta\zeta} K d^2\vec{r}. \qquad (3)$$

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The ETCC mechanism claims that in each surface patch there is neutralization tendency to cancel Δm_{eff}. This can be achieved by redistribution of existing TDs or via creation of additional pairs {defect, antidefect}. The latter could be realized if a structure exhibits strong enough localized elastic distortions [17,21-23]. This behavior arises because the TDs with positive (negative) topological charge are attracted to the regions exhibiting negative (positive) Gaussian curvature, as originally suggested in Refs. [22,23].

Modeling of Ordered Membranes

Below we focus on effectively 2D systems exhibiting in-plane nematic ordering. We review some of the existing different models of membrane ordering addressing spontaneously emergent nematic ordering.

Deviatoric Curvature Model

One could use the so called Deviatoric Curvature model [4,5,6,9] to describe the onset of nematic-type in-plane ordering due to anisotropic nature of membrane constituents. These are modelled by 2D anisotropic nano-plates of area a_0 [6, 24]. Their geometry is determined by the intrinsic curvature tensor

$$\underline{C}^{(m)} = C_1^{(m)}(\vec{e}_1^{(m)} \otimes \vec{e}_1^{(m)}) + C_2^{(m)}(\vec{e}_2^{(m)} \otimes \vec{e}_2^{(m)}). \quad (4)$$

Here $\{\vec{e}_1^{(m)}, \vec{e}_2^{(m)}\}$ and $\{C_1^{(m)}, C_2^{(m)}\}$ label eigenvectors and eigenvalues of the tensor, respectively, and $\otimes$ marks the tensorial product. The actual membrane shape is given by the effective curvature tensor field

$$\underline{C} = C_1(\vec{e}_1 \otimes \vec{e}_1) + C_2(\vec{e}_2 \otimes \vec{e}_2) \quad (5)$$

determined by the eigenvectors $\{\vec{e}_1, \vec{e}_2\}$ and eigenvalues $\{C_1, C_2\}$. The corresponding interaction energy density is expressed in terms of invariants formed by the mismatch curvature tensor [6,25]

$$\underline{M} = \underline{R}\,\underline{C}^{(m)}\underline{R}^{-1} - \underline{C}. \quad (6)$$

Here
$\underline{R} = (\vec{e}_1 \otimes \vec{e}_1 + \vec{e}_2 \otimes \vec{e}_2)\cos\theta + (\vec{e}_1 \otimes \vec{e}_2 - \vec{e}_2 \otimes \vec{e}_1)\sin\theta$
stands for the rotation tensor, and θ refers to the angle between $\vec{e}_1$ and $\vec{e}_1^{(m)}$ in the tangent plane of the membrane characterized by the surface normal $\vec{v} = \vec{e}_1 \times \vec{e}_2 = \vec{e}_1^{(m)} \times \vec{e}_2^{(m)}$. It follows [4-6]

$$w = \kappa((H - H^{(m)})^2 + D^2 - 2DD^{(m)}\cos(2\theta) + D^{(m)2}), (7)$$

where $\kappa > 0$ is the curvature modulus. The quantities $H = (C_1 + C_2)/2$, $H^{(m)} = (C_1^{(m)} + C_2^{(m)})/2$, $D = |C_2 - C_1|/2$, $D^{(m)} = |C_2^{(m)} - C_1^{(m)}|/2$ determine the mean curvature, mean intrinsic curvature, curvature deviator, and mean curvature deviator, respectively.

The ensemble average of the local orientational ordering is given by [26]

$$\eta = \langle\cos(2\theta)\rangle = \frac{\int_0^{2\pi}\cos(2\theta)e^{-wa_0\beta}d\theta}{\int e^{-wa_0\beta}d\theta} = \frac{I_1(x)}{I_0(x)}, \quad (8)$$

where I_n labels the Bessel function of n-th order and $x = 2\kappa a_0 DD^{(m)}\beta$.
Note that in the case of an isotropic intrinsic element Equation (7) yields

$$w = \kappa\left((H - C_0)^2 + D^4\right), \quad (9)$$

where $C_1^{(m)} = C_2^{(m)} = C_0$. Taking into account terms up to the 2nd order in the curvature tensor expansion and for $C_0 = 0$ one reproduces the "classic" Helfrich membrane curvature model.

Minimal Extrinsic Curvature Model

The nematic orientational ordering could appear also due to attached thin rod-like nano-objects, as for example BAR proteins [8]. The corresponding minimal model, where we assume that nano-objects are represented by thin flexible rod-like objects, may be given by the following expression for the free energy density $f = f_c + \phi f_n + f_{mix}$, where [8]

$$f_c = \frac{\kappa}{2}H^2, \quad (10a)$$

$$f_n = \frac{\kappa_n}{2}\left(C - C_n\right)^2, \quad (10b)$$

$$f_{mix} = \frac{k_B T}{a_0}\left(\phi\ln\phi + (1-\phi)\ln(1-\phi)\right) \quad (10c)$$

The first contribution f_c stands for the classical Helfrich spontaneous curvature term [2], where we assume isotropic intrinsic membrane constituents. The term f_n is the elastic curvature density contribution of the flexible

anisotropic rod-like nano-objects attached to the membrane surface. This energy term is weighted by the rigidity constant $\kappa_n > 0$. The constant C_n represents the locally enforced intrinsic curvature of the attached nano-object. The third term f_{mix} describes the configurational entropy contribution of the attached rod-like objects of surface area a_0. We refer to this model as the Minimal Extrinsic Curvature model [8].

The local curvature C, experienced by the attached flexible nano-rod, can be expressed as

$$C = C_1 \cos^2 \theta + C_2 \sin^2 \theta = H + D \cos(2\theta). \qquad (11)$$

Here θ describes the angle between the normal plane of the 1st principal curvature and the normal plane in which the nano-rod is lying.

The above described models are usually solved for axial membrane shape symmetry and are mainly used to determine qualitatively different membrane shapes [1,2,6,8]. A membrane shape is in the Cartesian coordinate frame $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ commonly parametrized as

$$r = \rho(s)\cos(\varphi)e_x + \rho(s)\sin(\varphi)\vec{e}_y) + \rho(s)\vec{e}_z. \qquad (12)$$

where $\varphi \in [0, 2\pi]$, and s describes the arc-length of the membrane profile in the (x,z) plane. Numerical details are described in [21]. The above model for orientational ordering of thin rod-like molecules can be further generalized by considering the orientational ordering of small plate-like flexible nanodomains embedded in the membrane [24] or flexible plate-like nano-objects attached to the membrane [27].

Landau-Type Mesoscopic Model

In order to emphasize role of orientational degrees of freedom the Landau-type modelling is commonly used [21,28,29]. To take into account the head-to-tail invariance it is necessary to introduce the tensor nematic order parameter [29]

$$\underline{Q} = \lambda(\vec{n} \otimes \vec{n} - \vec{n}_\perp \otimes \vec{n}_\perp). \qquad (13)$$

The quantity λ determines the *amplitude field*, the unit director field $\vec{n}$ determines mesoscopic local membrane in-plane orientation, $\vec{n} \cdot \vec{n}_\perp = 0$, and the membrane surface normal is defined as $\vec{v} = \vec{n} \times \vec{n}_\perp$.

In terms of invariants in terms of $\underline{C}$ and $\underline{Q}$ we express the free energy density $f = f_c + f_e^{(int)} + f_e^{(ext)}$, where we take into account only the most essential terms of our interest. Quantities f_c, $f_e^{(int)}$, $f_e^{(ext)}$ stand for condensation, intrinsic elastic and extrinsic elastic term, respectively. We express them as [21,29,30]

$$f_c = -ATr\underline{Q}^2 + B(Tr\underline{Q}^2)^2, \qquad (14a)$$
$$f_e^{(int)} = k_i Tr(\nabla_s \underline{Q})^2, \qquad (14b)$$
$$f_e^{(ext)} = k_e Tr(\underline{Q}\,\underline{C}^2). \qquad (14c)$$

The coefficients A and B are positive material constants, k_i and k_e stand for the representative intrinsic and extrinsic curvature elastic constants, $\nabla_s = (\underline{I} - v \otimes v)\nabla$ is the surface gradient operator [29] and ∇ represents the conventional gradient operator.

The model introduces several intrinsic (material dependent) length scales. The most important is the order parameter correlation length which we define as

$$\xi = \sqrt{\frac{k_i}{A}}. \qquad (15)$$

Membrane Nematic Order and Topological Defects

In the following we present some demonstrative results of presented mesoscopic continuum models focusing on in-plane membrane orientational ordering.

The Deviatoric Elastic model enforces orientational ordering if deviatoric terms D and D_m are different from zero. For simplicity we restrict to axially symmetric configurations. The ordering term in Equation (7) displays external-field like structure. The corresponding field strength monotonously increases with increasing value of D and D_m. Anisotropic membrane constituents orient along the direction for which eigenframes of tensors $\underline{C}$ and $\underline{C}^{(m)}$ coincide. The degree of nematic ordering $\eta = \langle \cos(2\theta) \rangle$ is plotted in Figure 2 as a function of dimensionless quantity $x = 2\kappa a_0 D D^{(m)}\beta$. The nematic ordering is absent if $\eta = 0$. Within this model nematic ordering is absent at umbilical points where $C_1 = C_2$. Therefore, the essential message that this model reveals is that anisotropic membrane constituents introduce an external field-type contribution in the interaction potential which is present in membrane areas

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where $C_1 \neq C_2$. Note that the model yields only information on local degree of ordering and does not take into account global topologically imposed constrains. Note that in general the distribution of the orientational ordering of the membrane components over the whole membrane and the global shape of the membrane are interdependent and determined by the minimum of the membrane free energy [5].
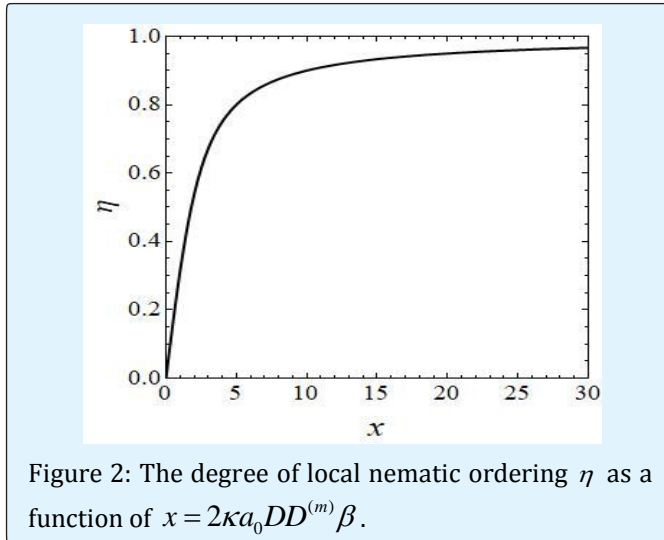

Figure 2: The degree of local nematic ordering η as a function of $x = 2\kappa a_0 DD^{(m)}\beta$.

We next consider the model described by the free energy density Eq.(10) within the Minimal Extrinsic Curvature model. In this treatment the nematic ordering arises due to the ordering of attached flexible rod-like nano-objects, to which we henceforth refer to as the *rods*. In this modelling one restricts to axially symmetric configurations of membranes. The variational parameters are the membrane shape, in-plane orientational ordering and spatially dependent concentration ϕ of the *rods*. We calculate the parameters by minimizing the free energy functional (see Eq.(10)) for a given average concentration ϕ_{ave} of nano-rods assuming constant membrane volume and its surface area.

Some representative results are shown in Figures 3 and 4. In Figure 3 we plot membrane shapes with superimposed orientational ordering for a fixed relative volume v and C_n on increasing the average concentration of *rods* ϕ_{ave} . As it can be seen the bottom part of membranes becomes increasingly spherical on expense of pronounced tubular extrusion. The rods align along the meridians. The Gauss-Bonnet and Poincare-Hopf theorems are obeyed: the total topological charge of structures equals two. The vortex-like topological defect of strength m=1 is localized at the poles of the structures.

Furthermore, on increasing C_n the rods become increasingly attracted to the tubular membrane extrusion. This is even more evident in Figures 4 where we increase the *rods*' imposed curvature C_n for a fixed average concentration ϕ_{ave} . Namely, in the tubular part of the membrane the mismatch between preferred curvature of *rods* and local curvature of membrane could be efficiently minimized. Consequently, the rods are expelled from the spherical part where curvature is significantly lower with respect to C_n. We remark that also in extreme case shown in Figure 4c the topological theorems are obeyed. A defect of strength m=1 resides at the top pole of the structure. The other m=1 TD residing in the bottom part of the structure is not visible because the nematic ordering there is absent.
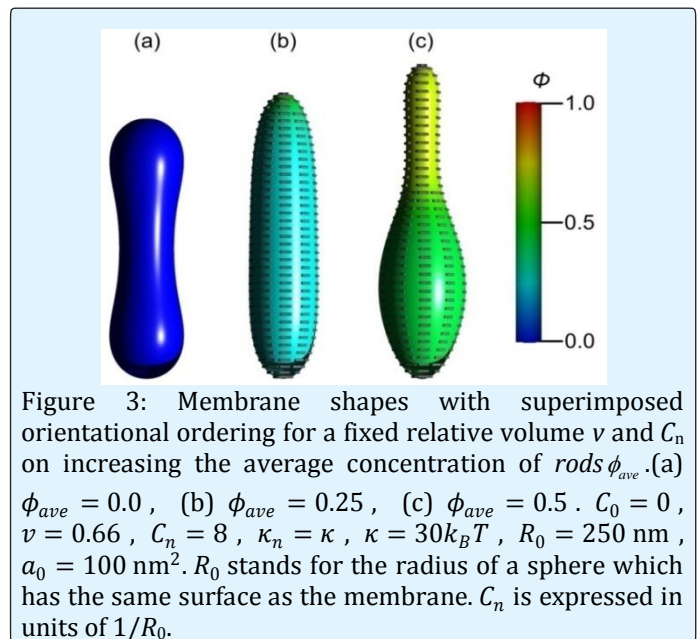

Figure 3: Membrane shapes with superimposed orientational ordering for a fixed relative volume v and C_n on increasing the average concentration of *rods* ϕ_{ave} .(a) $\phi_{ave} = 0.0$, (b) $\phi_{ave} = 0.25$, (c) $\phi_{ave} = 0.5$. $C_0 = 0$, $v = 0.66$, $C_n = 8$, $\kappa_n = \kappa$, $\kappa = 30 k_B T$, $R_0 = 250$ nm , $a_0 = 100$ nm^2. R_0 stands for the radius of a sphere which has the same surface as the membrane. C_n is expressed in units of $1/R_0$.
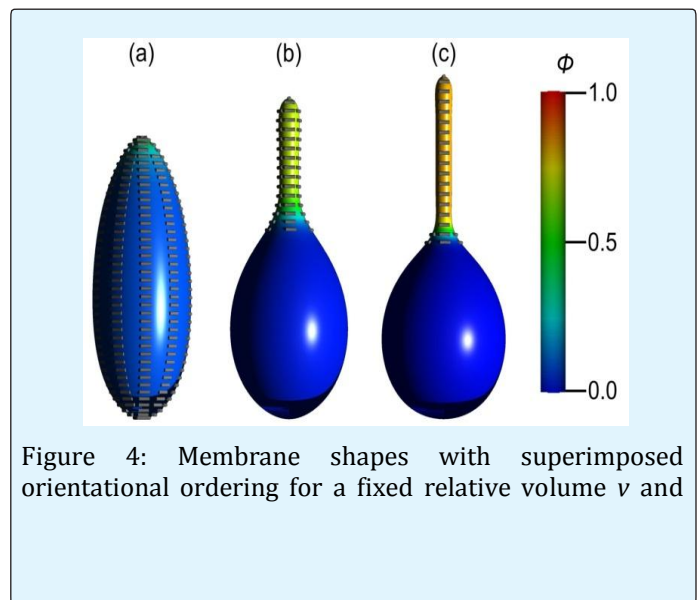

Figure 4: Membrane shapes with superimposed orientational ordering for a fixed relative volume v and

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ϕ_{ave} on increasing C_n. (a) $C_n = 10$, (b) $C_n = 12$, (c) $C_n = 14$. $C_0 = 0, v = 0.85, \phi_{ave} = 0.1, \kappa_n = \kappa, \kappa = 30 k_B T$, $R_0 = 250$ nm, $a_0 = 100$ nm^2.

Finally, we discuss the model given by Equation (14) which focuses mainly on nematic ordering for a given membrane shape. Note that the term weighted by the constant k_e mimics deviatoric-type term that was first introduced via the deviatoric elastic model [6]. In Figure 5 we show typical nematic ordering for a tubular-type nematic structure. In Figure 5a we depict structure in the $\{s, \varphi\}$ parameter plane, where s determines the membrane arc-length and v is the azimuthal angle in the (x,y) plane of the Cartesian coordinate system (x,y,z) of the axially symmetric membrane shape. In this modelling we allow solutions which break the axial symmetry. In the case shown three $m=1/2$ TDs are formed in the spherical part of the membrane exhibiting $\overline{K} > 0$, one $m=-1/2$ TD in the neck area where $\overline{K} < 0$, and $m=1$ TD at the bottom pole of the membrane, where $\overline{K} > 0$. The total charge equals two in accordance with Eq.(1). Furthermore, the distribution of TDs is in line with the ETCC mechanism, although the structure is not totally "topologically neutralized". For this purpose one would need stronger local distortions which would enable formation of an additional pair {defect, antidefect}={1/2,-1/2}. The resulting structure would possess four $m=1/2$, two $m=-1/2$ and one $m=1$ topological defects in the top spherical-like part, neck region, and bottom tubular extrusion, respectively.
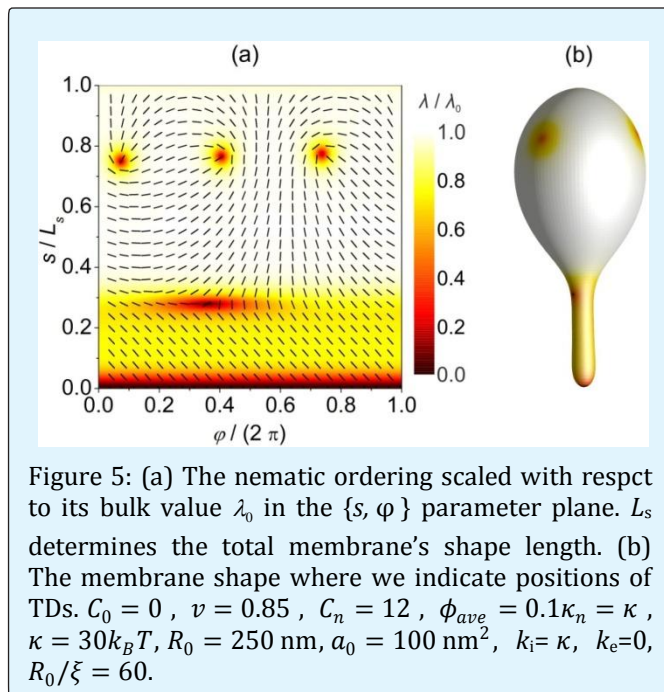

Figure 5: (a) The nematic ordering scaled with respct to its bulk value λ_0 in the $\{s, \varphi\}$ parameter plane. L_s determines the total membrane's shape length. (b) The membrane shape where we indicate positions of TDs. $C_0 = 0$, $v = 0.85$, $C_n = 12$, $\phi_{ave} = 0.1 \kappa_n = \kappa$, $\kappa = 30 k_B T$, $R_0 = 250$ nm, $a_0 = 100$ nm^2, $k_i = \kappa$, $k_e = 0$, $R_0/\xi = 60$.

Conclusions

We considered different mesoscopic-scale continuum models taking into account nematic-type in-plane membrane ordering.

The Deviatoric Elastic model [3-6] introduces local nematic ordering via anisotropic embedded or attached membrane constituents characterized by the two principal curvatures $C_1^{(m)} \neq C_2^{(m)}$. The local ordering tendency of orientational ordering field-like term is proportional to the local membrane curvature deviator $|(C_1 - C_2)/2|$. The term enforcing orientational ordering in the interaction potential adopts an external ordering field-type structure which is without direct interactions between the membrane components effective only in membrane parts where $C_1 \neq C_2$. Note that the external membrane curvature and the orientational ordering of the anisotropic membrane components over the whole membrane surface are actually interdependent and determined by the minimum of the total membrane free energy [5], i.e. the local membrane curvature is not only imposed by the intrinsic shape of the membrane components, but also by the global geometrical constrains and the minimum of the total membrane free energy.

The Deviatoric Elastic model thus gives both, the information on the local degree of the nematic ordering over the whole membrane surface and the equilibrium membrane shape [5].

Nematic-type order can be also enforced by rod-like attached nano-objects (rods). An example represent BAR family members [8] which impose locally preferred curvature due to their intrinsic curved shape. A hosting membrane responds by adopting a shape which compromises membrane local curvatures and local curvature imposed by rods. In the model the rods are allowed to move within the membrane and one imposes an axial symmetry. The resulting structures exhibits TDs. However, the imposed axial symmetry allows only TDs of integer strength $m=1$. The detailed in-plane nematic ordering description requires introduction of tensor nematic order parameter. This takes into account both orientation and amplitude of the ordering. We have shown that TD configuration obeys ETCC mechanism.

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A more detailed in-plane nematic ordering description requires introduction of the tensor nematic order parameter. The latter describes both orientation and amplitude of the orientational ordering. For a demonstrative purpose we show impact of membrane curvature on number and position of TDs in a membrane possessing a tubular extrusion. In spherically shaped membranes one expects typically four $m=1/2$ TDs which satisfies classical topological theorems. However, a tubular extrusion introduces a localized area where the Gaussian curvature is strongly positive and a neck region exhibiting negative Gaussian curvature. Such geometry enforces $m=1$ topological defect at the tip of a tubular extrusion. On the other hand a strong enough negative Gaussian curvature could trigger formation of pairs {*defect, antidefect*} bearing topological charges {1/2,-1/2}. Such configurations of TDs reflect tendency of "topological neutrality" of a total membrane structure in which spatially nonhomogeneous Gaussian curvature is "neutralized" by TDs.

To conclude, TDs might play important role in membranes. For example, TDs are sources of localized spatial nonhomogeneitie which could nucleate structural changes necessary for a specific membrane functional response. Furthermore, physics of TDs exhibits several universality. Therefore, lessons learned by studying TDs in biological cells could be useful also for other branches of physics.

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