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Basic Properties of Laplace Transformation in Mathematics

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Abstract

The main purpose of this research project is to discover the importance and results of Laplace transforms in the field of science and technology. Also try to highlight his role and valuable contributions. The goal is to emphasize the importance of mathematics in the presentation and study of complex systems and how they contribute to the development of science and technology.

In this article, we will learn the Laplace transform and its basics. The need for mathematics is increasing in modern life. In order to explain and prove their research on cognition, researchers in all scientific fields use different statistics, tools or models. The Laplace transform is one of the most important methods used by scientists and researchers to find answers to their problems.

Keywords: Laplace Transform; Mathematical Expression; Laplace Transform of Science and Technology

Introduction

The use of the Laplace transform played an important role in the development of technological knowledge. It provides a powerful tool for converting time-critical problems into quantitative problems, enabling researchers and engineers to efficiently solve many problems. Using the Laplace transform, it is possible to solve a number of problems, characterize system behavior and predict system results with exceptional accuracy. It was used approx. times in various fields such as engineering technology, basic sciences, mathematics and economics [1].

The Laplace transformation plays an important role in the management of technical systems. To analyze the operating system, it is necessary to perform the Laplace transformation of the differential function. Both the Laplace and the Laplace transform are used to control system dynamics. In this article, we will discuss in detail the definition; formulas, properties and application of the Laplace transform [2].

The Laplace transform provides an efficient method for solving initial value problems for constant-coefficient differential equations. However, the importance of the Laplace transform is not limited to this class of problems. Some understand the basics and the most important part of mathematics for engineers and mathematicians [3].

The Laplace transform is one of the most important mathematical tools that is widely used in various engineering applications. A similar transformation was first used by the French mathematician "Pierre Simon de Laplace" in his book

"Probability", and later this transformation was called the "Laplace transformation" [4].

The Laplace transform has some important properties that enable the transformation of a differential expression into an algebraic equation and simplify its solution, but it also provides a way to solve differential equations in cases where it is not possible to use the classical solution method. Functions that are stationary or defined by integrals. In addition to differential equations, using the Laplace transformation it is possible to solve belief-type equations, which are widely used in physics. Today, the Laplace transform is a very powerful tool for solving integral and differential equations. It has a significant linear property, so it is not possible to solve the general equation, but only one linear equation, but in practice it is more than enough. The basic idea is that this transformation reduces the given equation to an equation (algebraic or differential) that is easier to solve than the first one. Then, from the results of samples changed by different methods, we arrive at results for the first sample. Although it seems like a simple method at first glance, sometimes it is not easy to determine the Laplace transform. Now it is necessary to change some functions in order to "read" the answer to the first question from it. First we need to describe the time and the possible operation using the Laplace transform and we will show properties that will help us in quick calculations. Only then will we be able to know the true scope of the Laplace transform concept [3].

The Laplace transform is an integral transformation of a given derivative function with a real variable t to convert it into a complex function with a variable s. For t ≥ 0, let $f(t)$ be given and assume that the function satisfies certain conditions that will be stated later.

The Laplace transform $f(t)$, which is denoted by $L\{f(t)\}$ or $F(s)$ is defined by the Laplace transform formula:

$$F(s)=\int_0^{\infty} f(t)e^{-st}dt \qquad (1)$$

whenever the improper integral converges.

Standard notation: Where the notation is clear, we will use a capital letter to denote the Laplace transform, e.g. $L(f;\, s) = F(s)$.

The Laplace transform we have defined is sometimes called the one-sided Laplace transform. There is a two-sided version where the integral goes from −∞ to ∞. The Laplace transform is important in the field of control systems. To analyze the operating system, we need to perform the Laplace transform of the differential function. Both the Laplace transform and the Laplace transform are important in the analysis of dynamic control systems.

In many cases the Laplace transform can be used to solve initial value problems involving systems of differential equations. This method is used in the same way as when solving first value problems with higher order variables. However, for a system of differential equations, a system of algebraic equations is obtained by taking the Laplace transform of each equation. After the algebraic Laplace transform system is solved for each of unknown functions, the Laplace transform is used to find each unknown function to solve the system.

Laplace Transformation

The Laplace transform is a powerful mathematical tool with diverse applications in fields as diverse as engineering, physics, economics, and control theory. It is a mathematical technique that converts a time function into an s function and enables analysis of the function on a linear level. The Laplace transformation provides a method for solving differential equations and analyzing the behavior of linear proportional-invariant systems [5].

The Laplace transform was introduced to expand the category of frequency signals that can be interpreted by numerical methods. The problem of solving differential equations also manifests itself in the analysis and integration of the system as a whole. The analysis of the behavior of linear dynamics with time invariants is reduced to the problem of solving a system of differential equations with constant coefficients. The solution of these equations is simplified using the Laplace transformation [3].

The Laplace transform of the function $f(t)$ is defined as follows:

$$L\{f(t)\}=F(s)=\int_0^{\infty} x(t)e^{-st}dt \qquad (2)$$

where the complex variable s=δ+jω is called the complex frequency (frequency), and the real part of the complex variable s, Re{s}=δ, is chosen to ensure the convergence of the integral. The Laplace transform defined by the above expression is called the bilateral or two-sided Laplace transform. The two-sided Laplace transform is used for the analysis of power systems, as well as in certain applications in telecommunications and signal processing [6].

For the analysis of causal signals ($x(t) = 0$ for t<0), which are transmitted and processed by linear time-invariant causal (physically realizable), real systems, a fundamental role is played by the unilateral or one-sided Laplace transformation, which is defined by with the expression:

$$L\{f(t)\}=\int_{0^-}^{\infty} x(t)e^{-st}dt=F(s) \qquad (3)$$

Sometimes in the literature, the unilateral Laplace transform is defined as:

$$L\left\{f\left(t\right)\right\}=F\left(s\right)=\int_{0}^{\infty}x\left(t\right)e^{-st}dt \quad (4)$$

The difference between these two definitions is only in the lower limit of integration, which in the first definition is 0^-, and if the signal $x(t)$ does not contain the impulse delta signal $\delta(t)$ in the coordinate origin (t = 0), these two are basically identical. However, if the signal $x(t)$ contains an impulse delta function at the point t=0, it is necessary that the defining integral includes the entire delta impulse, so that the lower limit of integration must be $t=0^-$. Let's also note that the Laplace transformation was introduced in the theory of linear time-invariant systems primarily in order to determine the response of such a system to arbitrary excitation, and therefore also to impulse delta excitation. On the other hand, as shown earlier, the response of a static system consists of two parts (also known as the principle of superposition): the stimulus response (input or excitation signals) and the initial response, which represents the energy accumulated in the system before it occurs the excitement itself. If we assume that the time at which we set the stimulus for entering the system is t = 0, then the initial conditions must be defined immediately before applying the stimulus, that is, at $t = 0^-$. Therefore, the first definition of the Laplace transform will enable the determination of the response of the system both to the input signal and to the initial conditions.

The Laplace transform is a mathematical function that transforms the process of capturing the Laplace transform. Transform the Laplace curve function using complex numbers back to its original position.

This paper has a wide application in the analysis and solution of time-invariant and differential equations in engineering, science and mathematics [7].

By reproducing the behavior of work in time, the Laplace transform provides a deep and practical approach to real-time work. It serves as a fundamental tool in understanding and solving different systems and phenomena in different scientific disciplines.

$$L-1\left\{F\left(s\right)=f\left(t\right) \quad (5)$$

It is used to find the first time domain function from the Laplace transform. It is also used for solving differential equations and finding finite time solutions of systems defined in the Laplace domain [8].

The Laplace transform is used when different equations need to be converted into an algebraic form for easier calculation, study and analysis [9]. The Laplace transform is a fundamental transformation that transforms a function of a complex variable into a function of a real variable, usually time. The Laplace transform is a fundamental transformation commonly used to convert real-time differential equations to of polynomial equations in complex systems. It also helps solve linear differential equations because it transforms differential equations into simple algebraic operations and is an important part of applied mathematics, engineering, electrical engineering, and systems engineering.

The transformed functions and their solutions can be transformed back into the function in the original domain with the help of inverse integral transformations using inverse kernel functions, K-1(i,k). Such a transformation is called the inverse Laplace transform. To understand what the inverse Laplace transform is, it is necessary to understand the Laplace transform. If the function $f(t)$ is defined for all +ve values of t. The Laplace transform is denoted by the formula;

The inverse Laplace transform is a transformation opposite to the Laplace transform, which maps the given function $F(s)$ from the complex domain to the function $f(t)$ in the time domain. It is symbolically denoted as

$$f\left(t\right)=L^{-1}\left\{F\left(s\right)\right\} \quad (6)$$

It is mainly used for the character structure problem given by the equation i with the real coefficients of two polynomials in the variable s as a real function of the variables to the:

$$F\left(s\right)=\frac{P\left(s\right)}{Q\left(s\right)}\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0} \quad (7)$$

where the degree of the polynomial in the numerator is less than or equal to the degree of the polynomial in the denominator (m ≤ n).

The zeros of the polynomials $P(s)$ and $Q(s)$ are called the poles and zeros of the real rational function $F(s)$, respectively. Since these polynomials are with real coefficients, their zeros, that is, the poles and zeros of the complex character $F(s)$, can appear either as real or in conjugate-complex pairs, and can be simple and/or multiple.

$$Q\left(s\right)=s^n+a_{n-1}s^{n-1}+\cdots+a^1 s+a_0=0 \quad (8)$$

For finding the ILT, the poles of the function $F(s)$, i.e., are of particular interest the zeros of the polynomial $Q(s)$, i.e. the roots of the equation. This equation is called the characteristic equation, the characteristic polynomial polynomial.

Based on a known complex character, it is possible to determine the function (original) in the time domain by applying the inverse Laplace transformation. The original function $F(s)$ in the time domain $f(t)$ is determined using the following formula:

$$L^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma+j\omega}^{\sigma+j\omega} F(s)e^{st}ds; t > 0 \quad (9)$$

the previous relations are usually by the inverse Laplace transform, and together with the relation:

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt \quad (10)$$

It defines the so-called Laplace transform pairs. The fact that the functions $f(t)$ and $F(s)$ satisfy these two relations is often indicated in the literature by one of the following notations:

$$F(s) = L\{f(t)\}, x(t) = L^{-1}\{F(s)\}, f(s) \leftrightarrow F(s) \quad (11)$$

the formula of the inverse Laplace transform tells us that the signal $f(t)$ can be reconstructed on the basis of its Laplace transform pair, however, calculating this integral is very often a serious task and implies the so-called contour integration, which is studied in the theory of functions of complex variables. What we will mention here is that if the function $F(s)$ exists, then the inverse Laplace transform must be calculated according to the line σ=const. That line must belong to the area of convergence of the function $F(s)$, which means that the area of convergence must be such that there is a band of finite width and infinite length in it: σ1< Re{s} < σ2.

In general, the search for the Laplace transform is limited to the use of the definition formula, which requires the solution of a set of parameters that represent a complex processing technique. Solving this problem requires complex analytical skills, and actually solving a closed variable example (curvilinear solution) is something new. However, although we follow the theory of linear signals and systems in electrical engineering, i.e. to the in various fields such as automation, electronics, energy, communications, electrical circuits, signal processing, Laplace transform. All common signals of interest can be determined using methods to develop complex signals in some areas using the Laplace transform table and related properties. In general, finding the Laplace variable reduces to by means of a definitional formula that requires the solution of various elements that represent the work of a complex technique. Solving this problem requires complex analytical skills, and actually solving a closed variable example (curvilinear solution) is something new. But even though we follow the theory of linear signals and systems in electrical engineering. In various fields such as automation, electronics, energy, communications, electrical circuit theory, signal processing, the Laplace transform of all signals of interest and can be determined by methods used to develop complex structures of individual parts. The signal is obtained using the Laplace transform of a table of standard form and the corresponding properties of the Laplace transform [10].

The method of development of the complex character of the signal (Laplace transformation) to partial fractions can be applied only to the signal $f(t)$ whose complex character is a strictly rational function, i.e. represents the quotient of two polynomials with real coefficients:

$$F(s) = L\{f(t)\} = \frac{B_m(s)}{A_n(s)}\frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}; m < n \quad (12)$$

and such that the order of the polynomial in the numerator m is strictly less than the order n of the polynomial in the denominator A(s), i.e. m<n. For many real signals and systems, their Laplace transform takes the form of a strictly rational function. However, there is also a more common form of the equation in real life that represents a real function, where m = n, and the standard design is defined as a purely rational function. At the same time, in order to apply the technique of developing into fractions, it is first necessary to divide the polynomial into both a number and a difference, and the result of that division will be a constant factor, while the remaining part will enable rational calculation. . Some of these types of operations have been discontinued.

Ownership of Laplace Transformation

The Laplace transform is a fundamental transformation that connects the function $F(p)$ of a random variable with the function $f(t)$ of a real variable. With its help, the structure of the dynamic system is studied and its various and integral differences are resolved [11]. One of the properties of the Laplace transform, which determines its distribution in the mathematics of science and technology, is that it corresponds to simple relationships in the images of many relationships and functions in nature. Checking two functions in image space thus makes multiplication and linear differential equation algebraic.

Properties of linearity, similarity, differentiation, integration, delay, shift, convolution. Properties of the Laplace transform. We will mark with the $f(t)$, $g(t)$, ... originals, and through $F(p)$, $G(p)$, ... - their pictures.

Directly from the Integral Properties

$$F(p) = \int_0^{\infty} f(t)e^{-pt}dt, G(p) = \int_0^{\infty} g(t)e^{-pt}dt, \ldots \quad (13)$$

The properties of the Laplace transform greatly facilitate the task of finding images for a large number of different functions, as well as the task of finding originals from their images.

Example 1: Find function images:

a) sin ω*t*, cosω*t* ; b) sin 2*t* cos 5*t* ; c) $e^{-3t} \sin \pi t$.

The solution: a) Using the property of linearity and the formula obtained in example 2b), we find:

$$\sin\omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i} = \frac{1}{2i}\left(\frac{1}{p - i\omega} - \frac{1}{p + i\omega}\right) = \frac{\omega}{p^2 + \omega^2} \quad (14)$$

those $\sin\omega t = \frac{\omega}{p^2+\omega^2}$ .Similarly we get the formula $\sin\omega t = \frac{\omega}{p^2+\omega^2}$.

b) First, let's present the work sin 2*t* cos 5*t* as the difference

of sine: $\sin 2t\cos 5t = \frac{1}{2}(\sin 7t - \sin 3t)$, and then use the linearity and similarity properties:

$$\sin 2t\cos 5t = \frac{1}{2}\frac{1}{p^2+49} - \frac{1}{2}\frac{3}{p^2+9}.$$

c) Since $\sin\pi t = \frac{\pi}{p^2+\pi^2}$, then by the displacement property

$$e^{-3t}\sin\pi t = \frac{\pi}{(p+3)^2+\pi^2}.$$

Example 2: Find the original by its image

a) $F(p) = \frac{2p-5}{p^2-6p+11}$; b) $F(p) = \frac{1}{(p^2+\omega^2)^2}$; c) $F(p) = \frac{2p^2}{(p^2+1)^2}$.

The solution: a) Transform this fraction so that you can use the shift property

$$F(p) = \frac{2p-5}{p^2-6p+11} = \frac{2(p-3)+1}{(p-3)^2+2} = 2\cdot\frac{p-3}{(p-3)^2+(\sqrt{2})^2} + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{(p-3)^2+(\sqrt{2})^2}$$
$$= 2e^{3t}\cos\sqrt{2} + \frac{1}{\sqrt{2}}e^{3t}\sin\sqrt{2}t$$

(15)

b) Since $F(p) = \frac{1}{p^2+\omega^2}\cdot\frac{1}{p^2+\omega^2}$ *and* $\frac{1}{p^2+\omega^2} = \frac{1}{\omega}\sin\omega t$ that

$$F(p) = \int_0^t \frac{1}{\omega}\sin\omega\tau\cdot\frac{1}{\omega}\sin\omega(t-\tau)\,d\tau = \frac{1}{2\omega^2}\int_0^t (\cos\omega(2\tau - t) - \cos\omega t)\,d\tau =$$
$$\frac{1}{2\omega^2}\left(\frac{1}{2\omega}\cdot\sin\omega(2\tau-t)\Big|_0^t - \cos\omega t\cdot\tau\Big|_0^t\right) =$$
$$\frac{1}{2\omega^2}\left(\frac{1}{\omega}\sin\omega t - t\cos\omega t\right) = \frac{1}{2\omega^3}(\sin\omega t - \omega t\cos\omega t)$$

(16)

those $\frac{1}{(p^2+\omega^2)^2} = \frac{1}{2\omega^2} = (\sin\omega t - \omega t\cos\omega t)$.

c) Since

$$\frac{2p^2}{(p^2+1)} = 2p\cdot\frac{1}{p^2+1}\cdot\frac{p}{p^2+1}\ and\ \frac{1}{p^2+1} = \sin t, \frac{p}{p^2+1} = \cos t,$$

based on the formula we have

$$2p\cdot\frac{1}{p^2+1}\cdot\frac{p}{p^2+1} = 2\int_0^t \cos\tau\cos(t-\tau)\,d\tau + 0 = t\cos t + \sin t.$$

(17)

Application of Laplace Functions for Joint Differential Functions

The Laplace transform is used to directly solve two ordinary differential equations with constant coefficients and the same type of linear differential equation. It should be emphasized that one of the most important features of the Laplace transform in general is that the operations performed on images can be simplified more easily than the original operations. These conditions contribute to the efficient use and widespread use of analytical functions based on Laplace transformation in solving various problems in physics and technology. For example, differentiation and integration correspond to the simple operations of multiplication and division. Similarly, when different parameters are combined using the Laplace transform, the gains and losses are obtained according to known methods.

The Laplace transform can be used to solve various normals. It is used for differential, discrete and full-line systems. Now let's look at some examples of functions [12].

1^0 Solve an inhomogeneous linear differential equation with constant coefficients

$a_0 y^{(n)}(t) + a_1 y^{(n-1)}(t) + \ldots + a_{n-1}y'(t) + a_n y(t) = f(t)$, where it is understood that *f*(*t*) is the original.

Solution: Let it be $Y = Y(s) = L(y(t)), F(s) = L(f(t))$. Assume that the initial conditions are given

$y(0), y'(0), \ldots y^{(n-1)}(0)$. If we apply the Laplace transformation to the equation, we get

$$a_0\left(s^nY - s^{n-1}(0) - \ldots - y^{(n-1)}(0)\right) + a_1\left(s^{n-1}Y - s^{n-2}y(0) - \ldots - y^{(n-2)}(0)\right) + \ldots + a_{n-1}(sY - y(0)) + a_nY = F(s)$$

(18)

Introducing polynomials

$$P_n(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n \qquad (19)$$

$$Q_{n-1}(s) = a_0\left(s^{n-1}y(0) + \ldots + y^{(n-1)}(0)\right)a_1\left(s^{n-2}y(0) + \ldots + y^{(n-2)}(0)\right) + \ldots + a_{n-1}y(0) \qquad (20)$$

from the previous equation we get

$$Y(s)\frac{Q_{n-1}(s)}{P_n(s)} + \frac{F(s)}{P_n(s)} \qquad (21)$$

By applying the inverse Laplace transform, we get the solution

$$y(t) = L^{-1}\frac{Q_{n-1}(s)}{P_n(s)} + L^{-1}\frac{F(s)}{P_n(s)} = u_h(t) + y_p(t) \qquad (22)$$

where $yh(t)$ is the solution of the homogeneous equation, and $yp(t)$ is the particular solution of the homogeneous equation.

Conclusion

The Laplace transform is a versatile tool that simplifies the analysis of complex systems and transforms space-time problems into less complex algebraic problems, linear equations. The application has played an important role in science and technology. By simplifying the analysis and design of linear-invariant systems, the Laplace transform has played an important role in the advancement of technology in many areas, including control systems, signal processing, communications, and others.

The Laplace transform is a powerful mathematical tool that has proven important in many areas of research. For example, its ability to transform time functions into s functions makes it an invaluable tool for solving differential equations and analyzing linear time systems. Real-world applications of Laplace transforms are wide and varied, ranging from electrical and electronic control to economics and physics.

Based on a known complex character, it is possible to determine the function (original) in the time domain by applying the inverse Laplace transformation. The original function $F(s)$ in the time domain $f(t)$ is determined using the following formula:

$$L^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} F(s)e^{st}\,ds; t > 0 \qquad (23)$$

the previous relations are usually by the inverse Laplace transform, and together with the relation:

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt \qquad (24)$$

defines the so-called Laplace transform pairs. The fact that the functions $f(t)$ and $F(s)$ satisfy these two relations is often indicated in the literature by one of the following notations:

$$F(s) = L\{f(t)\}, x(t) = L^{-1}\{F(s)\}, f(s) \leftrightarrow F(s) \qquad (25)$$

the formula of the inverse Laplace transform tells us that the signal $f(t)$ can be reconstructed on the basis of its Laplace transform pair, however, calculating this integral is very often a serious task and implies the so-called contour integration, which is studied in the theory of functions of complex variables. What we will mention here is that if the function $F(s)$ exists, then the inverse Laplace transform must be calculated according to the line σ=const. that line must belong to the area of convergence of the function $F(s)$, which means that the area of convergence must be such that there is a band of finite width and infinite length in it: $\sigma 1 < \mathrm{Re}\{s\} < \sigma 2$.

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