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The Solution of A few Infinite Problems in the Mathematics

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Abstract

There are a lot of infinite problems in the mathematics which are very difficult for people including myself to deal with. The paper discusses a few typical infinite problems in the mathematics, it calculates out the sum of the infinite layers of the square root of '2"; it calculates out the result of the problem consisting of the infinite $\sqrt{2}$ power of $\sqrt{2}$; it demonstrates how to deal with the problem of infinite decimal; it calculates out the infinite sum consisting of $1-1+1-1+1-1+\ldots\ldots$; it calculates out the result of the infinite continued fraction; it calculates out the result of the problem consisting of the multiplication by infinite layers of $\sqrt{2}$; it proves an equality of infinite sum. The paper presents a method of solution which is commonly useful for most of the infinite problems in mathematics.

Keywords: Infinite Square Root; Infinite Power; Infinite Decimal; Infinite Continued Fraction; Infinite Multiplication; Infinite Sum

Abbreviation

IMO: International Mathematical Olympiad.

Introduction

There have been a lot of studies on the problems of IMO, this is principally due to that the IMO is very significant and important for the future of pupils in high schools, and in general, the problems of IMO are very difficult to solve. But in fact, there are a lot of other problems which are also very difficult for people and are usually recognized as puzzling problems by pupils of high schools and even the students in universities. For example, the infinite problems in the mathematics [1-10], for many cases of infinite problems, most people don't know how to consider those problems. In this paper, it will present a few infinite problems and the method of solving them.

The Method

With respect to any infinite problem in the mathematics, in order to solve it, we must change the infinite problem into a finite problem. Because there are various infinite problems in the mathematics, so there is no the only method to solve various infinite problems in the mathematics, it depends on the different problems, for example, if we want to calculate out $\sqrt{7\times\sqrt{7\times\sqrt{7\times\sqrt{7\times\ldots\ldots}}}}$, this is also an infinite problem in the mathematics, if let $x=\sqrt{7\times\sqrt{7\times\sqrt{7\times\sqrt{7\times\ldots\ldots}}}}$, it is easy to find that $x=\sqrt{7x}$, thus the problem has become a finite problem, it is easy to solve this equation and to get the result. In the following section, the paper will discuss a few other infinite problems in mathematics, and demonstrate the method of dealing with various infinite problems in the mathematics [2,3].

The Discussion and Result

The discussions on a few interesting infinite problems are given below:

- **Problem 1:** Calculating the infinite square root $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots}}}}$.

➢ **Solution:** Supposing $x=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots}}}}$, (1)

thus, equation (1) can be written as:

$$x=\sqrt{2+x}, \quad (2)$$

it arrives $x^2=2+x, x^2-x-2=0$, (3)

the solutions of equation (3) are given by: $x_1=-1, x_2=2$.

In accord with the problem, x must be positive $x=-1$, is evidently wrong, therefore, the solution of problem 1 is $x=x_2=2$ [4-6].

- **Problem 2:** Calculating the infinite power $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}$.

➢ **Solution**: Supposing $x=\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}$, (4)

from equation (4), it arrives $x=(\sqrt{2})^x=(2)^{\frac{1}{2}x}$, (5)

Equation (5) can be changed into $x^{-1}=(2)^{-\frac{1}{2}x}=(2^{-1})^{\frac{1}{2}x}$, (6)

from equation (6) it obtains

$$\left(\frac{1}{x}\right)^{\frac{1}{x}}=\left(\frac{1}{2}\right)^{\frac{1}{2}}, \quad (7)$$

therefore, $\frac{1}{x}=\frac{1}{2}$, (8)

The solution of the problem is $x=2$.

By the way, in fact, when $x=4$, equation (7) is also true, but using the mathematical induction, it can prove $x\le 2$. With respect to an arbitrary nth higher power, when n=1,

$$x=\sqrt{2}^{\sqrt{2}}\le\sqrt{2}^{2}=2, \quad (9)$$

supposing when n=k, $x\le 2$, thus, when n=k+1,

$$x=\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot^{\sqrt{2}(n=k+1)}}}}}\le\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}=2, \quad (10)$$

therefore, $x\le 2$, the solution of the problem can only be $x=2$.

- **Problem 3:** To calculate out $1-1+1-1+1-1+1-1-1+\ldots\ldots$

➢ **Solutions: Method 1:**
S=1-1+1-1+1-1+1-1+...... = (1-1)+(1-1)+(1-1)+(1-1)+...... = 0+0+0+0+0+0+...... = 0 (11)

➢ **Method 2:** S=1-1+1-1+1-1+1-1+.......=1-(1-1+1-1+1-1+1-1+......)= 1-S

therefore, 2S=1, (12)
it arrives: S=1/2, (13)

Thus, respect to the results of Equations (1) and (2), which one is right? Because we can't confirm whether the infinite 1-1+1-1+1-1+...... can be just infinite pairs (1-1)+(1-1)+(1-1)+......, so the correct result of this problem is S=1/2 [7-10].

- **Problem 4:** To change the infinite decimal $0.\dot{9}\dot{8}$ into a proper fraction.

Solution: Let $x=0.\dot{9}\dot{8}$, (14)
multiplying the two sides of equation (14) with 100, it arrives

$$100x=98+0.\dot{9}\dot{8}=98+x, \quad (15)$$

therefore, $x=0.\dot{9}\dot{8}=\frac{98}{99}$

- **Problem 5:** Compare 1 with $0.\dot{9}$, which one is bigger or smaller?

➢ **Solution:** At first, considering $0.\dot{9}$

$$\text{let } x=0.\dot{9}, \quad (16)$$

Multiplying the two sides of equation (16), it arrives

$$10x=9+0.\dot{9}=9+x, \quad (17)$$

It results in $9x=1, x=1$, (18)

therefore, $0.\dot{9}=1$, 1 is not bigger or smaller than $0.\dot{9}$.

- **Problem 6:** Calculating the infinitely continued fraction

$$\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots}}}$$

➢ **Solution:** Supposing $x=\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots}}}$, (19)

it can be written : $x=\frac{1}{2+x}$, (20)

namely $x^2+2x-1=0$, (21)

The solutions of equation (19) are given by:

$$x_1=-1-\sqrt{2}\,;\; x_2=-1+\sqrt{2} \quad , \tag{22}$$

in accordance with the problem, the solution must be positive, $x_1=-1-\sqrt{2}$ is evidently wrong! Therefore, the result of problem 6 is $x=x_2=-1+\sqrt{2}$.

- **Problem 7:** Calculating $\sqrt{2\times\sqrt{2\times\sqrt{2\times\sqrt{2\times\ldots\ldots}}}}$.

➢ **Solution:** Supposing x= $\sqrt{2\times\sqrt{2\times\sqrt{2\times\sqrt{2\times\ldots\ldots}}}}$, (23)

thus, from equation (23), it arrives :

$$x=\sqrt{2x} \;, \tag{24}$$

therefore $x^2=2x$, (25)

$$x\cdot(x-2)=0 \;, \tag{26}$$

the solution of equation (24) is $x_1=0, x_2=2$, but equation (18) is evidently not to be 0, so the result of problem 7 is $x=2$.

- **Problem 8:** Proving the equality $1^3+2^3+3^3+\ldots\ldots+n^3+\ldots\ldots=(1+2+3+\ldots\ldots+n+\ldots\ldots)^2$.

➢ **Solution:** Using the mathematical induction

When n=2, it is written; $1^3+2^3 = (1+2)^2$, the equality is true. Supposing when n=k (k is an arbitrary integer) the equality is also true, namely,

$$S_k^{\,2} = (1+2+3+\ldots\ldots+k)^2 = 1^3+2^3+3^3+\ldots\ldots+k^3 \;, \tag{27}$$

thus, When $n=k+1$, it is written:

$$\left[1+2+3+4+\ldots\ldots+k+(k+1)\right]^2=\left[S_k+(k+1)\right]^2=S_k^{\,2}+2S_k(k+1)+(k+1)^2, \tag{28}$$

Because the sum $S_k=k(k+1)/2$, substituting it into equation (28) it arrives

$$S_k^{\,2}+2S_k(k+1)+(k+1)^2=S_k^{\,2}+k(k+1)(k+1)+(k+1)^2=S_k^{\,2}+(k+1)^3, \tag{29}$$

Substituting equation (27) into equation (29), it results in

$$\left[1+2+3+4+\ldots\ldots+k+(k+1)\right]^2 = 1^3+2^3+3^3+\ldots\ldots+k^3+(k+1)^3, \tag{30}$$

Therefore, when n=k+1, the equality is still true, the problem 8 has been proven.

Conclusion

The paper discussed a few infinite problems in the mathematics and physics. From the discussion it concludes a key idea or a common way to solve those problems. With respect to an infinite problem, at first, it can be regarded as its first bit or first part and the other infinite part which can be recognised as the same infinite problem, for example, the problem 1 is recognised as $x=\sqrt{2+x}$, the x on the right of the equation is in fact the same as the x on the left of the equation. Thus, the puzzling problem becomes easy.

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