

Parameter Sensitivity of the Modified Green-Ampt Surface Sealing Infiltration Model

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Abstract

The Modified Green-Ampt Surface Sealing (MGASS) infiltration model considers real field infiltration process, wherein dispersed soil sediments, especially clay particles are present in the infiltrating water, and also accounts for the process of surface sealing. In view of the changes in the concentration of the soil sediments as infiltration proceeds, the various physical parameters in the equation are likely to change. Ponded infiltration studies were conducted in the field on three different soils to examine these changes in the model parameters, and how they influence the infiltration process. The results were compared with simulated infiltration using the Green-Ampt (G-A) MGASS infiltration models. The G-A model is often used to characterize infiltration process in soil hydrology. The key parameters in the MGASS model were the saturated hydraulic conductivity of the surface seal formed during the infiltration of water, hydraulic head, and moisture deficit; for the G-A model, the parameters were the saturated hydraulic conductivity of the soil surface, hydraulic head and moisture deficit. The saturated hydraulic conductivity (K_s) of the soil surface was found to be the most sensitive parameter in the G-A model. The MGASS equation, on the other hand was highly sensitive to changes in hydraulic head (h_f).

Keywords: Green-Ampt; Infiltration; MGASS model; Sensitivity; Surface seal

Introduction

The Green-Ampt (G-A) model is a widely used physically-based hydrological model due to its acceptable physical origin and computational simplicity [1 - 4]. The model assumes piston-type flow of soil water, and a step-

function for soil water content within the profile. There are two distinct soil water contents in this situation, the initial water content θ_i and the saturated water content θ_s near the soil surface [5]. The cumulative infiltration amount can be calculated by iteration process as presented in Equation (1):

$$F = K_s t + h_f (\theta_s - \theta_i) \ln \left(1 + \frac{F}{h_f (\theta_s - \theta_i)} \right) (1)$$

Where, *F* is the cumulative infiltration amount [L]; *K*_s is the saturated hydraulic conductivity [L/T]; *t* is the time from the start of the infiltration [T]; h_t is the pressure head [L]; is $\theta_s \cdot \theta_i$ the moisture deficit θ_d [L³/L³]; θ_i and θ_s are the initial and saturated moisture contents [L³/L³]

Since the development of the G-A model, it has gone through several series of modifications for various field conditions. For example, Mein and Larson [6] extended the model from ponded conditions to constant intensity conditions. Chu also applied this model to unsteady rainfall intensities. However, one major concern with regard to the applicability of the G-A model is the reason that infiltrating water always moves along with suspended soil sediments [7,8], hence, the introduction of the Modified Green-Ampt Surface Sealing (MGASS) infiltration model (Equation 2).

$$F = K_x(d)t + h_f(\theta_s - \theta_i) \ln\left(1 + \frac{F}{h_f(\theta_s - \theta_i)}\right)$$
(2)
$$K_x(d) = \frac{K_s}{C} d_*$$
(3)

where, *F* is the cumulative infiltration amount [L]; *D* is the mean particle diameter [L] of suspended sediments; *d*·is the dimensionless particle diameter of suspended sediments; K_x is the saturated hydraulic conductivity of the surface seal [L/T]; *t* is the time from the start of the infiltration [T]; h_f is the pressure head [L]; θ_s - θ_i is the moisture deficit θ_d [L³/L³]; θ_i and θ_s are the initial and saturated moisture contents [L³/L³]

The MGASS equation also incorporates the process of surface sealing and is capable of estimating the thickness of the surface seal formed as a result of the deposition of the suspended soil sediments, which is otherwise, difficult or almost impossible to measure experimentally [9]. Thus, the presence the soil particle phase in infiltrating water could greatly affect the infiltration process in the field. Additionally, just like all simulation models, prediction uncertainties arising from factors such as violation of underlying assumptions on which the model was developed, uncertainty in model parameters [10], and sensitivity of model parameters to changes are also expected in the field application of the MGASS equation.

Detailed reviews on the historic development of infiltration theory including the classic solutions based on the Richards' equation have been provided [11]. This equation for soil moisture movement plays a very important role in contemporary engineering science, applied hydrology [12], soil science and agronomy. During model simulations, it is always imperative to estimate and report intrinsic prediction uncertainties, which usually result from the violation of assumptions inherent in the model, and also uncertainty in the model parameters [10]. However, it is very important to consider the fundamental assumptions on which these models are formulated since the unselective applications of theories developed for humid hydrology do not always carry out accurately in arid environments due to slaking of soil aggregates and dispersion of clays [9,13]. No single model best meets all possible requirements, hence, in depth knowledge and understanding of model performance under different conditions are always required before valuable decisions and recommendations on the application of a model can be made. The choice of a model, would therefore, depend on the type of application, expected level of physical/mathematical rigour, and user preference [10].

Significant attempts have, thus, been made towards the quantification and reduction of prediction errors in infiltration models, however, most of these studies are characterized by limited assessment of model structure. Sensitivity analysis could serve as additional tool showing the relationship between model input factors and output variables in hydrologic modeling processes. This extensive analysis is key to identifying potential deficiencies in model structures and formulation, explain and correct the lack of fit of hydrological models, provide guidance for model reduction and parametrization, analyze the information content of available observations, and describe the subspace of the original control space driving predictive uncertainty [14]. Hence, the objective of the present study was to evaluate the parameter sensitivity of the MGASS model in comparison with the G-A model, and to determine their optimal parameters that would improve the utility of the models for simulating infiltration.

Materials and Methods

Field Infiltration Measurements

Field infiltration studies were conducted on three different soils, namely, Stagni-Dystric Gleysol (SDG), Plinthi Ferric Acrisol (PFA) and Plinthic Acrisol (PA). Ponded infiltration measurements were conducted using a single ring infiltrometer of 30 cm diameter and 20 cm height [9,15]. The process involved inserting infiltrometer rings vertically into the soil to a depth of 10 cm with the

Tuffour HO, et al. Parameter Sensitivity of the Modified Green-Ampt Surface Sealing Infiltration Model. J Agri Res 2019, 4(3): 000223.

aid of a mallet and a plank. Water at a pressure head of 5 cm was gently added in the extended cylinder, and maintained with water from a 1000 ml measuring cylinder. Infiltration was measured for 60 minutes. Initial measurements were conducted at regular time intervals of 30 seconds for five minutes after ponding when infiltration was very fast for the determination of sorptivity. The time interval was increased to 1, 3 and 5 minutes as infiltration slowed down towards the steady state. Analyses of infiltration parameters were done as discussed in Tuffour et al. [9].

Measurement of Saturated Hydraulic Conductivity

Saturated hydraulic conductivity was conducted on undisturbed soil cores in the laboratory using the modified falling head permeameter as described by Tuffour [13], and Bonsu and Laryea [16]. Intact soil cores were saturated for 24 hours; after which they were placed on gravels supported by a plastic sieve, and placed in a sink. Water was gently added to give hydraulic head in the extended cylinder and the fall of the hydraulic head (h_t) on the soil surface was measured as a function of time (t) using a water manometer with a 5-meter scale. The K_s was estimated by the standard falling head equation:

$$K_s = \left(\frac{aL}{At}\right) \ln\left(\frac{h_o}{h_t}\right)$$
(5)

where, *a* is the surface area of the cylinder [L²]; *A* is the surface area of the soil [L²]; *h*_o is the Initial hydraulic head [L]; *L* is the length of the soil column [L]; is the hydraulic head after a given time t [L]. Rewriting equation (1), a regression of $\ln \left(\frac{h_o}{h_t}\right)$ on *t* with slope $b = K_s \left(\frac{A}{La}\right)$ was obtained. Since *a*=*A* in this particular case, *K*_s was simply calculated as:

 $K_s = bL(6)$

Sensitivity Analysis

Condition Number

Parameter perturbation was employed to provide a measure of the sensitivity of each model parameter [17]. Each parameter was varied by $\pm 25\%$, ± 50 , and $\pm 75\%$ of its mean value. The parameters considered herein were K_s , h_f and θ_d for the G-A model (Equation 1), and $h_{j_i} \theta_d$ and K_x for the MGASS model (Equation 2). The sensitivity of the cumulative infiltration amount was evaluated with each of these estimates varied about its mean value while all other parameter values were held constant. The

condition number (*CN*) was calculated at six specific times, i.e., 5, 10, 15, 30, 45 and 60 minutes for each case.

$$CN_p = \frac{\bar{p}\,\Delta n}{n\,\Delta p} \ (7)$$

Where, CN_p is the condition number [dimensionless] for the parameter p; \overline{p} is the mean measured value for the parameter p; n is the dependent variable; Δp is the change in the independent variable; Δn is the change in the dependent variable. In the present study, n represented the cumulative infiltration amount (*F*), and p, a specific parameter of each model.

Sensitivity Index

Sensitivity of model outputs to the changes in input parameters was described according to Lenhart et al. [18]. and Ravazzani et al. [19]. Sensitivity of the model output to changes in the input parameters was described by a dimensionless sensitivity index [18]. Mathematically, a variable *y* is dependent on a parameter *x* by a partial derivative, $\partial y/\partial x$ which is numerically approximated by a finite difference. Assume y_0 is the model output computed from an initial value x_0 of the parameter *x*, which is varied by $\pm \Delta x$ yielding $x_1 = x_0 \cdot \Delta x$ and $x_2 = x_0 + \Delta x$, yielding corresponding y_1 and y_2 , respectively. The finite approximation of the partial derivative $\partial y/\partial x$ becomes:

$$I' = \frac{y_2 - y_1}{2\Delta x} \ (8)$$

The dimensionless index was obtained by normalization as follows:

$$I = \frac{(y_2 - y_1)/y_o}{2\Delta x/x_o}$$
(9)

The sign (i.e., positive or negative) of the index describes the direction of reaction of the model (i.e., if an increase of the parameter results in an increase of the output variable and a decrease of the parameter to a decrease of the variable, or inversely). For this study, Δx was fixed at 25% irrespective of the range of variation of tested parameters. Accordingly, the sensitivity of the model output was ranked according to Lenhart et al. [18] as presented in Table 1.

Class	Index	Sensitivity
Ι	$0.00 \le I < 0.05$	Small to negligible
II	$0.05 \le I < 0.2$	Medium
III	$0.2 \le I < 1.00$	High
IV	$ I \ge 1.00$	Very high

[|]I| = Dimensionless sensitivity index **Table 1:** Sensitivity index classes

4

Results and Discussion

Results of sensitivity analysis on cumulative infiltration amount predicted from G-A and MGASS equations are presented in Table 2. The results show that the G-A equation is very sensitive to K_s and moderately sensitive to θ_d and h_f . The MGASS equation, on the other hand showed high sensitivity to K_x , and negligible sensitivity to θ_d and h_f .

Danamatan		G-A Equation		MGASS Equation				
Parameter	SDG	PFA	PA	SDG	PFA	PA		
$\theta_d (\mathrm{cm}^3/\mathrm{cm}^3)$	Ι	Ι	Ι	II	II	II		
h_f (cm)	Ι	Ι	Ι	III	III	III		
K_s (cm/min)	III	IV	IV	-	-	-		
K_x (cm/min)	-	-	-	Ι	Ι	Ι		

SDG = Stagni-Dystric Gleysol; PFA = Plinthi Ferric Acrisol; PA = Plinthic Acrisol; *Gray shades describe negative values **Table 2:** Sensitivity index classes of infiltration with G-A and MGASS equations.

Based on the form of the equations it was apparent that increasing each of the parameters, other than F, would result in an increase in F. The responses of each

model parameter to perturbation are given by their varied condition numbers in Tables 3a – b for the G-A and MGASS and models, respectively.

		SDG	PFA			РА			
ΔB (%)	K _s	h _f	θ_d	K _s	h _f	θ_d	K _s	h _f	$\boldsymbol{\theta}_{d}$
-25	0.26	3.75	0.24	0.76	3.75	0.32	0.99	3.75	0.46
-50	0.18	2.50	0.29	0.51	2.50	0.36	0.66	2.50	0.47
-75	0.088	1.25	0.34	0.25	1.25	0.39	0.33	1.25	0.48
Base value	0.35	5.00	0.19	1.02	5.00	0.30	0.32	5.00	0.45
+25	0.44	6.25	0.14	1.27	6.25	0.15	1.65	6.25	0.050
+50	0.53	7.50	0.088	1.53	7.50	0.18	1.97	7.50	0.061
+75	0.61	8.75	0.037	1.78	8.75	0.22	2.30	8.75	0.071

 ΔB = Change in base parameter value; SDG = Stagni-Dystric Gleysol; PFA = Plinthi Ferric Acrisol; PA = Plinthic Acrisol **Table 3a:** Changes in base parameter value in Green & Ampt equation.

		SDG	PFA			РА			
ΔΒ (%)	K _x	h _f	θ_d	K _x	h _f	θ_d	K _x	h _f	θ_d
-25	4.1E-4	3.75	0.24	9.8E-4	3.75	0.33	1.1E-3	3.75	0.46
-50	2.7E-4	2.50	0.29	6.5E-4	2.50	0.36	7.5E-4	2.50	0.47
-75	1.4E-4	1.25	0.34	3.3E-4	1.25	0.39	3.7E-4	1.25	0.48
Base value	5.5E-5	5.00	0.19	1.3E-3	5.00	0.30	1.5E-3	5.00	0.45
+25	6.8E-4	6.25	0.14	1.6E-3	6.25	0.27	1.9E-3	6.25	0.050
+50	8.2E-4	7.50	0.088	2.0E-3	7.50	0.24	2.2E-3	7.50	0.061
+75	9.6E-4	8.75	0.037	2.3E-3	8.75	0.21	2.6E-3	8.75	0.071

 ΔB = Change in base parameter value; SDG = Stagni-Dystric Gleysol; PFA = Plinthi Ferric Acrisol; PA = Plinthic Acrisol **Table 3b:** Changes in base parameter value in the MGASS equation.

Green-Ampt Model Sensitivity

Table 4 shows a summary of the sensitivity of G-A equation to *F*. Figures 1 – 3 show the trend of cumulative infiltration amount to changes in K_s , θ_d and h_f . Cumulative

infiltration amount was significantly sensitive to changes in the θ_d , h_f , and the K_s in respect of the condition numbers (Table 3a) but less sensitive to changes in θ_d and h_f .

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ΔΒ (%)	SDG			PFA			PA		
	K _s	h _f	θ_d	K _s	h _f	$\boldsymbol{\theta}_{d}$	K _s	h _f	θ_d
-25	18.06	22.93	23.67	50.54	46.94	46.14	66.03	84.53	85.88
-50	12.80	22.48	23.98	35.27	43.94	46.46	46.29	83.11	85.99
-75	7.53	21.92	24.26	19.99	42.77	46.77	26.55	81.42	86.09
Base value	23.32	23.32	23.32	46.74	46.74	46.74	85.77	85.77	85.77
+25	28.58	23.65	22.91	81.08	46.80	45.46	105.51	86.88	85.67
+50	33.84	23.94	22.41	96.35	47.31	45.09	125.25	87.89	85.57
+75	39.10	24.21	21.78	111.62	47.97	44.71	144.99	88.82	85.46

ΔB = Change in base parameter value; SDG = Stagni-Dystric Gleysol; PFA = Plinthi Ferric Acrisol; PA = Plinthic Acrisol **Table 4**: Sensitivity of cumulative infiltration amount with G-A equation.



Figure 1a: Effects of changes in saturated hydraulic conductivity in the G-A model on cumulative infiltration amount in the Stagni-Dystric Gleysol.









Figure 2a: Effects of changes in saturated hydraulic conductivity in the G-A model on cumulative infiltration amount in the Plinthi Ferric Acrisol.



Figure 2b: Effects of changes in pressure head in the G-A model on cumulative infiltration amount in the Plinthi Ferric Acrisol.







Figure 3a: Effects of changes in saturated hydraulic conductivity in the G-A model on cumulative infiltration amount in the Plinthic Acrisol.



Figure 3b: Effects of changes in pressure head in the G-A model on cumulative infiltration amount in the Plinthic Acrisol.



Figure 3c: Effects of changes in soil moisture content in the G-A model on cumulative infiltration amount in the Plinthic Acrisol.

MGASS Model Sensitivity

Table 5 shows a summary of the sensitivity of MGASS

equation to cumulative infiltration amount. Figures 4 – 6 show the responses of *F* to changes in K_x , h_f and θ_d .

ΔΒ (%)	SDG			PFA			PA		
	K _x	h _f	θ_d	K _x	h _f	θ_d	K _x	h _f	θ_d
-25	2.30	1.92	2.66	4.79	3.93	5.14	6.88	5.66	7.01
-50	2.29	1.47	2.97	4.77	2.94	5.46	6.86	4.25	7.12
-75	2.28	0.91	3.25	4.75	1.76	5.77	6.83	2.55	7.22
Base value	2.31	2.31	2.31	4.81	4.81	4.81	6.90	6.90	6.90
+25	2.31	2.64	1.90	4.82	5.59	4.46	6.92	8.01	6.81
+50	2.32	2.93	1.40	4.84	6.31	4.09	6.95	9.02	6.70
+75	2.33	3.20	0.77	4.86	6.97	3.71	6.97	9.95	6.60

ΔB = Change in base parameter value; SDG = Stagni-Dystric Gleysol; PFA = Plinthi Ferric Acrisol; PA = Plinthic Acrisol **Table 5**: Sensitivity of cumulative infiltration amount with MGASS equation.



Figure 4a: Effects of changes in surface seal saturated hydraulic conductivity in the MGASS model on cumulative infiltration amount in the Stagni-Dystric Gleysol.



Figure 4b: Effects of changes in the pressure head in the MGASS model on cumulative infiltration amount in the Stagni-Dystric Gleysol.



Figure 4c: Effects of changes in the soil moisture content in the MGASS model on cumulative infiltration amount in the Stagni-Dystric Gleysol.



Figure 5a: Effects of changes in surface seal saturated hydraulic conductivity in the MGASS model on cumulative infiltration amount in the Plinthi Ferric Acrisol.







Figure 5c: Effects of changes in the soil moisture content in the MGASS model on cumulative infiltration amount in the Plinthi Ferric Acrisol.



Figure 6a: Effects of changes in surface seal saturated hydraulic conductivity in the MGASS model on cumulative infiltration amount in the Plinthic Acrisol.



Figure 6b: Effects of changes in the pressure head in the MGASS model on cumulative infiltration amount in the Plinthic Acrisol.





The mathematical accuracy, robustness and applicability of the sensitivity analysis are verified through one-dimensional vertical infiltration. In general, the sensitivity analysis showed that the G-A model is highly sensitive to K_s compared to h_f and θ_d (Table 2) in all the three soils. Comparison of the condition numbers revealed that K_s contributed more to F, than h_f and θ_d . A similar observation was made by Turner, who compared five infiltration equations and their field validations and found that the G-A equation is much more sensitive to changes in K_s than in h_f and θ_d , respectively. Thus, changes in *F* varied linearly with changes in K_s [20]. On the other hand, the MGASS equation was highly insensitive to changes in K_x ; a change in K_x resulted in insignificant

change in *F*. However, changes in *F* resulting from changes in h_f were high (Table 2). Sensitive parameters show strong differences between the two models.

Altogether, sensitivity analysis was successful in identifying the most important parameter in both infiltration equations, although different results with regard to parameter sensitivities were obtained. Thus, insights on which parameter(s) contribute most to the outputs of the models have been achieved. Classification of parameters were different, which could on the most part have resulted from the differences of the sensitivity index around the class boundaries. Sensitivity analysis has proven to be a valuable tool for the assessment of the input parameters with respect to their impact on model output, and can thus, be very essential model validation and reduction of uncertainty [20].

Conclusion

A comparison of two infiltration equations has been presented in this study. Results showed that, the MGASS equation was highly sensitive to changes in h_f followed by θ_d and K_x . The Green-Ampt equation, on the other hand responded extremely high to changes in the K_s than h_f and $\theta_{d_{t}}$ respectively. Thus, hydraulic head, moisture content, and saturated hydraulic conductivity of the soil surface were found to be the key parameters influencing infiltration of water in soils in the MGASS and G-A equations, respectively. Thus, the output of a particular model not only depends on the structure of the model, but also on the input parameters. The current study was successful in identifying the most important input parameter (i.e., $h_{\rm f}$) in the MGASS equation, which should be given key consideration during the calibration and validation of the MGASS equation.

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