



# Flow Analysis and Heat Transfer of Nanofluid Flow in Different Geometries: A Review with Focus on Recent Development

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## Abstract

In a thermo-dynamical system loss of energy takes the centre of attention. Laws of thermodynamics stated that energies of the system cannot be lost, but this energy could be engaged to perform useful work, or wastefully lost in form of rises in temperature of the system. It is eminent to control the factors which act in rising values of loss in energy. Nanofluids uses nanosized particles with very high thermal conductivity uniformly distributed in base fluids which increases the conductivity of the base fluid ridiculously. Nanofluid play a vital role in reducing the loss of energy and improve heat conduction. An effort has been made in this paper is to carry out an extensive review of the literature regarding Nanofluid in recent years. Some basic components and properties of nanofluids are deeply elaborated in this article. Preparation of nanofluids perform a very significant role in recent decades. The new advanced results in nanofluids helps the reader to clarify their concepts are argued using two major dynamical models.

**Keywords:** Nanofluid; Shape effects; Thermo-physical properties; Mathematical models

## Introduction

Water and oil are most commonly used for heat transportation due to their fluidity, but low heat transfer character is an area of concern for such fluids. Whereas, the heat conductivity of the metals are much higher as compared to fluids, so it is of common interest to produce a class of fluid which can conduct heat or electricity like metals or their oxides. Nanofluids are by added nanosized particle with very high thermal conductivity in some base fluid [1-11]. Nowadays the technique involving both nanofluid and porous media finds considerable attention from many researchers and great demand from industry-based thermal systems. The logic behind it is that the surface area in contact with fluid in porous medium increases [12-16], while nanoparticles dispersed in nanofluid upsurge the effective thermal conductivity leading to the dramatic enhancement

of the efficiency of typical industrial thermal systems. It is of vital importance to know the factor which reduces the thermal efficiency. According to laws of thermodynamics energy of the system remains to conserve, but can be converted into other forms for the utility [17-20]. More commonly we say all the energies of the system are spent in doing work or to augment the temperature of the body. Rise in use of nanofluids is one way to reduce the loss of energy. Increasing the heat transfer rate of heat transfer equipment is an ever-lasting topic in thermal engineering.

The motivation of this review paper is to emphasize on the researchers to pay attention to the basic understanding of heat transfer enhancement due to nanofluids and its components. The knowledge presented in this paper is of signifies in the engineering appliances which needs the efficient heat transfer heat transfer mediums.

## Nanofluids

Industrial advancement in recent years boosted a need for efficient heat transfer and cooling process both at the micro or larger scale. Conventional fluids like air, water, or oils have smaller heat conductivity when compared with metallic solids.

The idea of adding solid particles in the liquid to enhance the conductivity of liquids has been floated about a century ago when Maxwell established mathematical relations of the electrical conductivity of solid particles. But, these experiments and theoretical studies have been conducted with particles of size millimeter or micrometer. These particles tend to settle down quickly or form an aggregation. Also, they resist flow and cause a drop in pressure. To add to that a large number of particles are needed to improve the thermal conductivity of the fluid. Considering the adverse effects and limited advantage of the solid-liquid suspension these fluids can't hit the limelight.

With the advanced modern technology particles of a size, less than 100 nm are prepared of the metals and their oxides with better thermal conductivity, mechanical and magnetic performance base on the requirement of the system. These nanoparticles are uniformly dispersed in in Newtonian and non-Newtonian base fluids. These are stable suspend of nanoparticles with typical dimensions of the order of 10 nm. The term "Nanofluid" was first used by Choi [21].

The theme to develop such fluids is to achieve the best thermal performance of the material which can be deformed or transported like fluids with a very small concentration of particles. It is important to learn the mechanism of enhancement as to know why a drastic increase in heat transfer character of nanofluid is observed. Figure 1 shows the increase in literature on nanofluid in last five year. Many scientists swiftly mounting nanofluids [22-46] proposed a different mechanism behind this adversity. Also, defines new mathematical models for the properties and flow behavior of nanofluids.

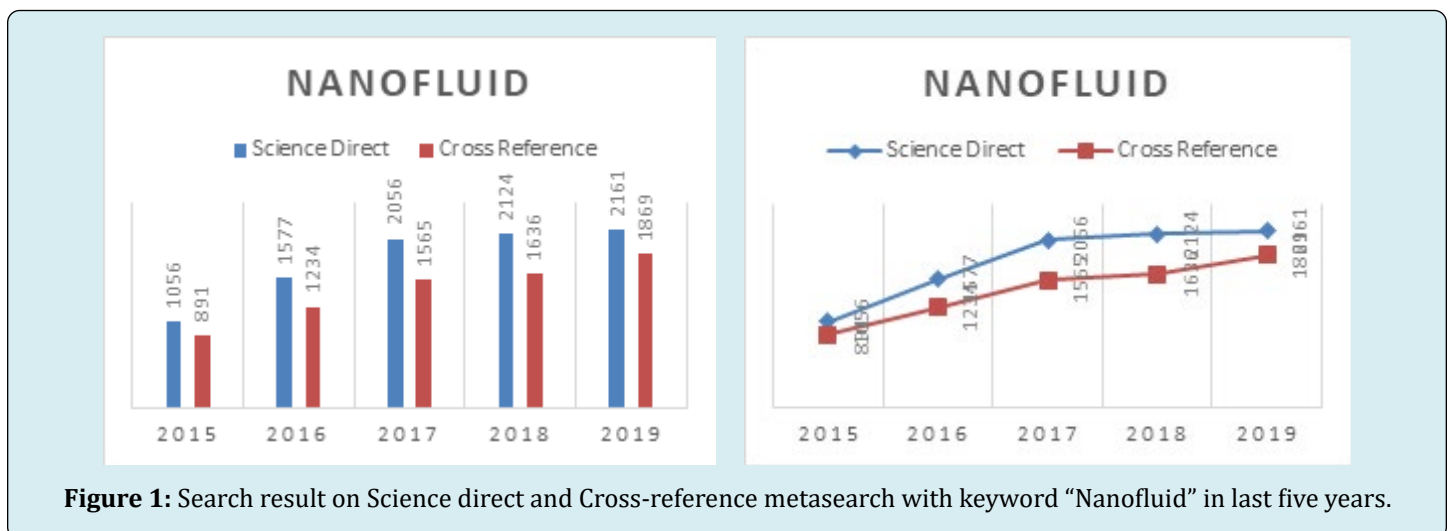


Figure 1: Search result on Science direct and Cross-reference metasearch with keyword "Nanofluid" in last five years.

### Components of Nanofluid

Nanofluids are composed of Nanosized particles, base fluids and surfactants which forms a heterogeneous mixture.

**Nanoparticles:** Many metals and there oxides, Nitrides and carbides are used as nanoparticles. All possess properties to enhance thermal conductivity. Materials used for nanoparticles can be

- Metal (Al, Cu, Ag, Au etc).
- Metal carbides/Nitrides/oxides (SiC, SiN, AlN, Al<sub>2</sub>O<sub>3</sub>, CuO etc)
- Nonmetals (carbon nanotubes)
- Hybrid Nanofluid(includes multiple nanoparticles used in fixed proportions)

**Base Fluids:** Usually, the base fluid used for these nanoparticles as a carrier are fluids with low heat transfer rate or non-conductor of electricity (also called ferrofluids) in their pure form. Ideally, it is required to enhance such properties of the fluid. To generator a smart coolant or smart fuel in a weightless environment. Some example of base fluids are

- Water
- Ethylene
- Oils
- Biofluids
- Polymer solutions

**Surfactants:** Again comes the problem of stability of nanofluids i.e. agglomeration and clogging of nanofluids in

microchannels. Here, surfactant comes in play also known as dispersants. The surfactant is an economical way to increase the stability of nanofluid. The selection of the proper surfactant for given nanofluids is a key issue. Surfactant depends on the base fluids and nanoparticles used to prepare a nanofluid. Materials like can sometimes be used as the surfactant. Additionally, it is observed that with the inclusion of surfactants

- Sodium dodecyl benzoic sulfate
- Silica

### Preparation of Nanofluids

Nanofluids are prepared using two processes known as Two-step method and single step method.

**Two-step process:** In a two-step, process nanoparticles are fabricated as dry powders using different chemical or physical processes. Then, the powder is uniformly dispersed in the base fluid and finally, high power magnetic field or shear mixing is applied to get a homogenous nanofluid. As powder manufacturing of these metallic particles is already a developed process and its techniques and manufacturing at commercial scale are already continuing in the industry this process is considered to be a cheaper process to get nanofluids at a commercial level. But, particles due to the sheer number and large surface area, they have a tendency to aggregate and provides unstable behaviour.

**Single step process:** Considering the difficulties, many techniques are employed to simultaneously make and disperse nanosized particles in base fluids in a so-called single step process. This process reduces the chances of agglomeration of nanoparticles by dispersing the particles as soon as developed and avoids process like dying and storing etc. But on the hand these methods cannot synthesize nanofluids for commercial scale and lifted the cost of such fluid to a very high level, alternative chemical processes are deployed for one step processes which are improving rapidly.

### Physical Properties of Nanofluids

Almost a decades of extensive experimental and theoretical research on nanofluid couldn't still unveil all the hidden wonders of nanofluids. Also, a lack of agreement on mathematical and theoretical aspects of nanofluids and many possibilities of preparing nanofluids results in existence of a huge number of correlations defining physical and chemical properties of nanofluids. Factor like size, the shape of nanoparticles, base fluid, its pH value, surfactants, the thickness of the layer of surfactant and method for the production of nanofluids etc are the factor which affects the results and consequently make it hard to model the exact behaviour of any nanofluid.

**Shape and size of Nanofluid:** Many shapes and sizes of nanoparticles can be prepared using different techniques. The shapes and sizes affect the Nanofluid's thermal ability. Shapes like needle, platelets, brick, cylinder, rod and wire are more commonly used in literature other than traditional spherical particles. Yu and Choi [47] assumed ellipsoidal particles suspended in liquid with semi-axes of  $\alpha$ ,  $\beta$  and  $\sigma$ . The equation satisfying the solid ellipsoidal is

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\sigma^2} = 1 \quad (1)$$

For a needle shape  $\alpha \gg \beta = \sigma$ , for disc  $\alpha = \beta \gg \sigma$  and for sphere  $\alpha = \beta = \sigma$ . The Maxwell's macroscopic effective medium theory (EMT) [48] was extended by Hamilton and Crosser [49]. Some of the studies on the shape of the nanoparticles are described in Table 1. Size of nanoparticles an interesting relationship also. with increase an of size nanofluids conductivity decrease as the problems like clogging, agglomerations resurfaces. Many experimental studies show this results (See table [1]). Empirical shape factor  $n$  was introduces as  $n=3/\psi$ , here  $\psi$  is the sphericity which can be calculated by dividing surface of the sphere and other surfaces.

Numerous investigators have since reported remarkable physical and mechanical properties of carbon. Carbon nanotubes illustrates a growing number of applications of carbon nanotubes (CNTs) in analytical chemistry. The structure of carbon nanotubes is first briefly summarized followed by a description of the characterization methods such as STM, TEM, neutron diffraction, X-ray diffraction, X-ray photoelectron spectroscopy, infrared and Raman spectroscopy. Carbon nanotubes due to their specific atomic structure have interesting chemical and physical properties according to those of graphite and diamond. The characterization methods of carbon nanotubes which are most employed today.

From unique electronic properties and a thermal conductivity higher than diamond to mechanical properties where the stiffness, strength and resilience exceeds any current material, carbon nanotubes offer tremendous opportunities for the development of fundamentally new material systems. The mechanical properties of carbon nanotubes, combined with their low density, offer scope for the development of nanotube-reinforced composite materials. The natural convection boundary layer flow along a vertical cone with variable wall temperature under the presence of magneto-hydrodynamics is investigated. Vibration analysis of single-walled carbon nanotubes (SWCNTs) based on Love's thin shell theory has been investigated along with five sort of boundary conditions with

three different shapes.

The unique relationship between SWNT atomic structure and electronic properties, and the richness of structures observed in both purified and chemically etched nanotube samples are discussed. A more detailed picture of SWNT electronic band structure is developed and compared with experimental tunneling spectroscopy measurements. The experimental and theoretical investigations of localized

structures, such as bends and ends in nanotubes, are presented. Last, quantum size effects in nanotubes with lengths approaching large molecules are discussed. The implications of these studies and important future directions are considered. Carbon nanotubes improved the ammonium biosensor response, linearity range of biosensor, detection limit of sensor; Multiwalled carbonnanotubes increased the response signal [50-60].

Effects	Reference	Particle type	Particle sizes	Base fluid	Relevant information
Size effects	Masuda, et al. [61]	Al <sub>2</sub> O <sub>3</sub>	13 nm	Water	Enhancement(Volume fraction): 4.3% Enhancement(Thermal conductivity): 33%
	Eastman, et al. [62]	Al <sub>2</sub> O <sub>3</sub>	33 nm	Water	Enhancement(Volume fraction): 4.3% Enhancement(Thermal conductivity): 9%
	Lee, et al. [63]	Al <sub>2</sub> O <sub>3</sub>	38 nm	EG/Water	Enhancement(Volume fraction): 5.0%(EG/ water), Enhancement(Thermal conductivity): 18%(EG) and 12%(Water)
	Wang, et al. [64]	Al <sub>2</sub> O <sub>3</sub>	28 nm	EG Water	Enhancement(Volume fraction): 5% Enhancement(Thermal conductivity): 17%(EG) and 14%(Water)
	Xie, et al. [65,66]	Al <sub>2</sub> O <sub>3</sub>	60.4 nm	Water EG	Enhancement(Volume fraction): 5%(EG/ Water) Enhancement(Thermal conductivity): 22%(Water) and 29%(EG)
	Das, et al. [67]	Al <sub>2</sub> O <sub>3</sub>	38 nm	Water	Enhancement(Volume fraction): 4% Enhancement(Thermal conductivity): 8%
	Putra, et al. [68]	Al <sub>2</sub> O <sub>3</sub>	131 nm	Water	Enhancement(Volume fraction): 4% Enhancement(Thermal conductivity): 25%
	Wen and Ding [69]	Al <sub>2</sub> O <sub>3</sub>	37-56 nm	Water	Enhancement(Volume fraction): 1.6% Enhancement(Thermal conductivity): 10%
	Nara, et al. [70]	Al <sub>2</sub> O <sub>3</sub>	40 nm	Water EG	Enhancement(Volume fraction): 0.5%(Water/EG) Enhancement(Thermal conductivity): 34%(Water), 5%(EG)
	Chon, et al. [71]	Al <sub>2</sub> O <sub>3</sub>	11, 47, 150 nm	Water	$\mu_{150nm} \leq \mu_{47nm} \leq \mu_{11nm}$

Shape Effects	Li and Peterson [72]	Al <sub>2</sub> O <sub>3</sub>	36, 47 nm	Water	Enhancement(Volume fraction): 6% Enhancement(Thermal conductivity): 28%
	Krishnamurthy, et al. [73]	Al <sub>2</sub> O <sub>3</sub>	20 nm	Water	Enhancement(Volume fraction): 1% Enhancement(Thermal conductivity): 16%
	Zhang, et al. [74]	Al <sub>2</sub> O <sub>3</sub>	20 nm	Water	Enhancement(Volume fraction):5% Enhancement(Thermal conductivity): 15%
	He, et al. [75]	TiO <sub>3</sub>	95, 145, 210 nm	Water	$\mu_{210\text{nm}} \leq \mu_{145\text{nm}} \leq \mu_{95\text{nm}}$
	Li and Peterson [76]	Al <sub>2</sub> O <sub>3</sub>	36, 47 nm	Water	$\mu_{47\text{nm}} \leq \mu_{36\text{nm}}$
	Anoop, et al. [77]	Al <sub>2</sub> O <sub>3</sub>	45, 150 nm	Water	$\mu_{150\text{nm}} \leq \mu_{45\text{nm}}$
	Vajjha and Das [78]	ZnO	29, 77 nm	EG-Water	$\mu_{77\text{nm}} \leq \mu_{29\text{nm}}$
	Patel, et al. [79]	Al <sub>2</sub> O <sub>3</sub>	11, 45, 150 nm	EG, oil, water,	$\mu_{150\text{nm}} \leq \mu_{45\text{nm}} \leq \mu_{11\text{nm}}$
	Teng, et al. [80]	Al <sub>2</sub> O <sub>3</sub>	20, 50, 100 nm	Water	$\mu_{100\text{nm}} \leq \mu_{50\text{nm}} \leq \mu_{20\text{nm}}$
	Akbulut, et al. [81]	ZnS	Sphere, rod, and wires		The increased steric contribution of the nanostructures to the overall surface interaction
	Timofeeva, et al. [82]	Al <sub>2</sub> O <sub>3</sub>	Platelet, blade, cylinder, and brick	EG/water	Elongated particles and agglomerates resulted in higher viscosity at the same volume fraction due to structural limitation of rotational and transitional Brownian motions
	Cui, et al. [83]	Cu	Sphere and cylinder		Enhancement of Sphere(Viscosity): 14.8% Enhancement of Cylinder(Thermal conductivity): 20.31% Enhanced micro-convection of the cylindrical shape from rotational motion, which increased thermal conductivity
	Ghosh and Pabi [84]	Cu	Cylinder		Increased contact area with increase of aspect ratio of nanoparticle induced higher rate of heat transfer during the collision
	Pilkington and Briscoe [85]	N/A	N/A	N/A	Aspect ratio of the nanostructures has an effect on the equilibrium forces mediated by nanofluids that can affect the viscosity of nanofluids

	Ooi and Popov [86]	Cu	Sphere and spheroid		Enhancement(Viscosity): 40-603% Enhancement(Thermal conductivity): 32-151% Estimated by the HeC model and the Mueller et al.'s model
	Ferrouillat, et al. [87]	SiO2 ZnO	Sphere and banana Polygonal and rod	Water	Following the Timofeeva et al.'s analysis (2009). Enhancement is lower than predicted by the HeC model. Banana shape nanoparticles have larger surface area in contact with stabilizing chemicals than that of spherical ones

**Table 1:** Size and Shape effects of nanoparticles in nanofluids.

**Density of Nanofluid:** Density is defined as mass per unit volume. Vajjaha and Das [88] perform a number of experiments with ethylene glycol and water base fluids. The relationship is defined as

$$\rho_{nf} = \phi \rho_p + (1 - \phi) \rho_f \quad (2)$$

The mathematical equation for the density of the two-phase mixture of solid in a liquid is adopted by Pak and Cho [89] and verify the result for  $Al_2O_3$ .

**Specific Heat capacity:** Mathematically, specific heat capacity at constant pressure is the amount of heat required to raise the temperature of the 1 Kg fluid through 1 K. A correlation for the specific of nanofluid is calculated by Pak and Cho [89] for  $Al_2O_3$ -water nanofluid is

$$c_{nf} = \phi c_p + (1 - \phi) c_f \quad (3)$$

Here,  $c_f$  is calculated ASHRAE Handbook [90] as

$$c_f = 4.2483T + 1882.4 \text{ for } 293K \leq T \leq 363K \quad (4)$$

The results were improved by Buongiorno [91] by including density in it. The expression becomes

$$(\rho c)_{nf} = \phi(\rho c)_s + (1 - \phi)(\rho c)_f \quad (5)$$

**Viscosity:** The viscosity of the fluid with the spherical particle is calculated by Einstein [92] as

$$\frac{\mu_{eff}}{\mu_f} = 1 + 2.5\phi \quad (6)$$

It is established that no theoretical model as yet can explain the viscosity of nanofluid completely but there are numerous correlations available defining the viscosity for some restricted conditions. The viscosity of Nanofluids depends mainly on the concentration of nanoparticles, but also, the temperature of the fluid, shape, size and density of the particles. Many theoretical and experimental investigations lead to different viscosities in different scenarios and improvement is overseen in the current models. Many review papers show a comprehensive review of effective viscosity of nanofluids [93-97]. Table 2 shows some of the theoretical and experimental models for the viscosity of nanofluids.

Model	Reference	Year	Correlation	Relevant information
	Saito [98]	1950	$\frac{\mu_{eff}}{\mu_f} = 1 + \frac{2.5}{(1 - \phi)} \phi$	Spherical rigid particles
	Brinkman [99]	1952	$\frac{\mu_{eff}}{\mu_f} = \frac{1}{(1 - \phi)^{2.5}}$	Brownian motion Very small particles Spherical particles

Theoretical	Lundgren [100]	1972	$\frac{\mu_{eff}}{\mu_f} = \frac{1}{1 - 2.5\phi}$	Valid for high moderate particle concentrations Dilute concentration of spheres
	Batchelor [101]	1977	$\frac{\mu_{eff}}{\mu_f} = 1 + 2.5\phi + 6.2\phi^2$	Rigid and spherical particles
	Drew and Passman [102]	1999	$\frac{\mu_{eff}}{\mu_f} = 1 + 2.5\phi$	Brownian motion Isotropic structure $\phi < 5.0$ vol%
	Wang, et al. [64]	1999	$\frac{\mu_{eff}}{\mu_f} = 1 + 7.3\phi + 123\phi^2$	Cu/water, Au, CNT, graphene Al <sub>2</sub> O <sub>3</sub> /water
	Tseng and Lin [103]	2003	$\frac{\mu_{eff}}{\mu_f} = 13.47 \exp(35.98\phi)$	Al <sub>2</sub> O <sub>3</sub> /ethylene glycol TiO <sub>2</sub> /water nanofluids
	Maiga, et al. [104]	2005	$\frac{\mu_{eff}}{\mu_f} = 1 + 7.3\phi + 123\phi^2$	Al <sub>2</sub> O <sub>3</sub> /water nanofluids
	Maiga, et al. [105]	2005	$\frac{\mu_{eff}}{\mu_f} = 1 - 0.19\phi + 306\phi^2$	Al <sub>2</sub> O <sub>3</sub> /ethylene glycol nanofluids
	Song, et al. [106]	2005	$\frac{\mu_{eff}}{\mu_f} = 1 + 56.5\phi$	SiO <sub>2</sub> /water nanofluids
	Koo and Kleinstreuer [107]	2005	$\mu_{\text{Brownian}} = 5 \times 10^4 \beta \rho_f \phi \sqrt{\frac{\kappa_B T}{2 \rho_p r_p} \left[ \frac{(-134.63 + 1722.3\phi) + (0.4705 - 6.04\phi) \frac{T}{T_0}}{1} \right]}$	CuO/water nanofluids

Experimental	Kulkarni, et al. [108]	2006	$\ln \mu_{eff} = A \left( \frac{1}{T} \right) - B,$ $A = 20587\phi^2 + 15857\phi + 1078.3$ $B = -107.12\phi^2 + 53.54\phi + 2.8715$	$0.05 \leq \phi \leq 0.15$ $d_p = 29 \text{ nm}$ CuO-water $278 \leq T \leq 323 \text{ K}$ Shear rate = 100 1/s
	Buongiorno [91]	2006	$\frac{\mu_{eff}}{\mu_f} = 1 + 5.45\phi + 108.2\phi^2$ $\frac{\mu_{eff}}{\mu_f} = 1 + 39.11\phi + 533.9\phi^2$	TiO <sub>2</sub> /water nanofluids Al <sub>2</sub> O <sub>3</sub> /water nanofluids
	Chen, et al. [109]	2007	$\frac{\mu_{eff}}{\mu_f} = 1 + 10.6\phi + 112.36\phi^2$	TiO <sub>2</sub> /ethylene glycol nanofluids
	Nguyen, et al. [110]	2007	$\frac{\mu_{eff}}{\mu_f} = 0.904 \exp(0.1483\phi) \quad d_p = 47 \text{ nm}$ $\frac{\mu_{eff}}{\mu_f} = 1 + 0.025\phi + 0.015\phi^2 \quad d_p = 36 \text{ nm}$ $\frac{\mu_{eff}}{\mu_f} = 1.475 - 0.319\phi + 0.051\phi^2 + 0.009\phi^3$ $d_p = 26 \text{ nm}$	Al <sub>2</sub> O <sub>3</sub> /water nanofluids CuO/water nanofluids



Namburu, et al. [111]	2007	$\text{Log}(\mu_{eff}) = A \exp(-BT)$ $A = 1.8375\phi^2 - 29.643\phi + 165.56$ $B = 4 \times 10^{-6}\phi^2 - 0.001\phi + 0.0186$	CuO/(60:40)
Grag, et al. [112]	2008	$\frac{\mu_{eff}}{\mu_f} = 1 + 11\phi$	EG/water nanofluids Cu/ethylene glycol nanofluids
Masoumi, et al. [113]	2009	$\mu_{eff} = \mu_f + \frac{\rho_p v_B d_p^2}{72C\delta}, \delta = \sqrt[3]{\frac{\pi}{6\phi} d_p}$	Al <sub>2</sub> O <sub>3</sub> /water nanofluids
Duangthongsuand Wongwises [114]	2009	$\frac{\mu_{eff}}{\mu_f} = a + b\phi + c\phi^2$ <p> <math>a = 1.0226, b = 0.0477, c = -0.0112</math> for <math>T = 15^\circ\text{C}</math>  <math>a = 1.0130, b = 0.0920, c = -0.0177</math> for <math>T = 25^\circ\text{C}</math>  <math>a = 1.0180, b = 0.1120, c = -0.0177</math> for <math>T = 35^\circ\text{C}</math> </p>	TiO <sub>2</sub> /water nanofluids
Chandrasekar, et al. [115]	2010	$\frac{\mu_{eff}}{\mu_f} = 1 + b \left( \frac{\phi}{1 + \phi} \right)^n$ $b = 1631, n = 2.8$	Al <sub>2</sub> O <sub>3</sub> /water nanofluids

Vajjha, et al. [78]	2010	$\frac{\mu_{eff}}{\mu_f} = A \exp(C\phi)$ $A = 0.9197, C = 22.8539$ $\mu_b = A \exp(B/T)$ $A = 0.555 \times 10^{-3}, B = 2664$	CuO/(60:40)
Corcione [116]	2011	$\frac{\mu_{eff}}{\mu_f} = \frac{1}{1 - 34.87(d_p/d_f)^{-0.3} \phi^{-1.03}}$ $d_f = 0.1 + \left( \frac{6M}{N\pi\rho_{f,0}} \right)^{1/3}$	EG/water nanofluids SiO <sub>2</sub> /ethanol nanofluids

**Table 2:** Summary of the studies on the theoretical and experimental models effective viscosity of nanofluids.

**Thermal conductivity:** Like viscosity, thermal conductivity also has no reliable model that effectively define all fluids in all conditions many correlations both theoretical and experimental are available in literature defining thermal conductivity (table 3). It is seen through experiments that thermal conductivity enhances the presence of nano-sized particles in fluid drastically hence an increase in thermal conductivity of the fluid is evident. Thermal conductivity shows its variation with change in the density of both fluid and particles, shape, size, concentration, heat conductivity etc. Maxwell [48] defined conductivity base on EMT for spherical particles as

$$k_{eff} = \frac{(kp+2kf)+2(kp-kf)\phi}{(kp+2kf)-(kp-kf)\phi} \quad (7)$$

Hamilton and cross [49] incorporated shape factor and defines thermal conductivity as

$$k_{eff} = \frac{(kp+(n-1)kf)+(n-1)(kp-kf)\phi}{(kp+(n-1)kf)-(kp-kf)\phi} \quad (8)$$

Vajjha, et al. [78] displays effects of density

$$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)} + 5 \times 10^4 \beta \rho_p c_p \sqrt{\frac{\kappa_B T}{\rho_p D}} f(T, \phi) \quad (9)$$

Corcione [116] defined the dependence of thermal conductivity on the fluid temperature.

$$\frac{k_{eff}}{k_f} = 1 + 4.4 \text{Re}^{0.4} \text{Pr}^{0.66} \left( \frac{T}{T_{fr}} \right)^{10} \left( \frac{k_p}{k_f} \right)^{0.03} \phi^{0.66} \quad (10)$$

Some review papers display effective viscosity and thermal conductivity recently [93-97].

Experimental and theoretical models are developed some of such models are shown in Table 3.

Reference	Correlation	Relevant information
Bruggemann [117]	$\frac{k_{eff}}{k_f} = \frac{1}{4} \left[ (3\phi - 1) \frac{k_p}{k_f} + (2 - 3\phi) \right] + \frac{k_f}{4} \sqrt{\Delta}$ $\Delta = \left[ (3\phi - 1)^2 \left( \frac{k_p}{k_f} \right)^2 + (2 - 3\phi)^2 + 2(2 + 9\phi - 9\phi^2) \frac{k_p}{k_f} \right]$	Spherical particle Spherical particles Applicable to high concentrations
Wasp [118]	$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)}$	Spherical and non-spherical particles, Micro-dimensions Various particle shapes, Hamilton and Crosser's model with $n = 3$ .  Shape factor is unity.
Davis [119]	$\frac{k_{eff}}{k_f} = 1 + \frac{3(k-1)}{(k-2) - \phi(k-1)} \left[ \phi + f(k)\phi^2 + O\phi^3 \right]$	$f(k) = 2.5$ for $k = 10$ $f(k) = 0.5$ for $k = \infty$
Lu and Lin [120]	$\frac{k_{eff}}{k_f} = 1 + a\phi + b\phi^2$	Spherical and non-spherical particles For $k = 10$ : $a = 2.25, b = 2.27$ $k = \infty$ : $a = 3.00, b = 4.51$

Xue [121]	$9 \left( 1 - \frac{\phi_p}{\lambda} \right) \frac{k_{eff} - k_f}{2k_{eff} + k_f} + \frac{\phi_p}{\lambda} \frac{k_{eff} - k_{c,x}}{k_{eff} + B_{2,x} (k_{c,x} - k_{eff})} + \frac{\phi_p}{\lambda} 4 \frac{k_{eff} - k_{c,y}}{2k_{eff} + (1 - B_{2,x})(k_{c,y} - k_{eff})} = 0$	Spherical particles Nano-layer
Bhattacharya, et al. [122]	$\frac{k_{eff}}{k_f} = \frac{k_p}{k_f} \phi + (1 - \phi)$ $k_p = \frac{1}{\kappa_B T^2 V} \sum_{j=0}^n Q(0) Q(j\Delta T) \Delta T$	Brownian dynamics
Koo and Kleinstreuer [123], [124]	$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)} + 5 \times 10^4 \beta \rho_p c_p \sqrt{\frac{\kappa_B T}{\rho_p c_p}} f(T, \phi)$	CuO/ethylene glycol CuO/oil Considered surrounding liquid traveling with randomly moving nanoparticles
Prasher, et al. [125]	$\frac{k_{eff}}{k_f} = \left( 1 + A \text{RePr}^{0.333} \phi \right) \left( \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)} \right)$	Effect of convection of the liquid near the particle included $A$ is constant Nanospheres
Xue [126]	$\frac{k_{eff}}{k_f} = \left( \frac{1 - \phi + 2\phi \left( \frac{k_p}{k_p + k_f} \right) \ln \left( \frac{k_p + k_f}{2k_f} \right)}{1 - \phi + 2\phi \left( \frac{k_f}{k_p + k_f} \right) \ln \left( \frac{k_p + k_f}{2k_f} \right)} \right)$	Nanospheres with interfacial shell

Xue and Xu [127]	$\left(1 - \frac{\phi_p}{\alpha}\right) \frac{k_{eff} - k_f}{2k_{eff} + k_f} + \left[ \left(\frac{\phi_p}{\lambda}\right) \left\{ \frac{(k_{eff} - k_{shell})(2k_{shell} + k_p) - \alpha(k_p - k_{shell})(2k_{shell} + k_{eff})}{(2k_{eff} + k_{shell})(2k_{shell} + k_p) + 2\alpha(k_p - k_{shell})(k_{shell} - k_{eff})} \right\} \right] = 0$	Spherical particles Nano-layer
Xie, et al. [128]	$\frac{k_{eff} - k_f}{k_f} = 3\Theta\phi_T + \frac{3\Theta^2\phi_T^2}{1 - \Theta\phi_T}$	Low particle loadings Nano-layer
Li and Peterson [129]	$\frac{k_{eff} - k_f}{k_f} = 0.764\phi + 0.0187(T - 273.15) - 0.462$ $\frac{k_{eff} - k_f}{k_f} = 3.761\phi + 0.0179(T - 273.15) - 0.307$	Al <sub>2</sub> O <sub>3</sub> /water nanofluids CuO/water nanofluids
Buongiorno [92]	$\frac{k_{eff}}{k_f} = 1 + 2.92\phi - 11.99$	TiO <sub>2</sub> /water nanofluids
Timofeeva, et al. [130]	$k_{NF} = (1 + 3\phi)k_f$	Al <sub>2</sub> O <sub>3</sub> /water nanofluids
Avsec and Oblak [131]	$\frac{k_{eff}}{k_f} = \frac{k_p + (n-1)k_f - (n-1)(1+\beta)^3\phi(k_f - k_p)}{k_p + (n-1)k_f + (1+\beta)^3\phi(k_f - k_p)}$	$n = (3/\psi) -$ empirical shape factor

Chandrsekhar, et al. [132]	$\frac{k_{eff}}{k_f} = \left[ \frac{k_p + (n-1)k_f - (n-1)(1+\beta)^3 \phi (k_f - k_p)}{k_p + (n-1)k_f + (1+\beta)^3 \phi (k_f - k_p)} \right]$ $+ \frac{C\phi(T-T_0)}{\mu k a^4}$	Al <sub>2</sub> O <sub>3</sub> /ethylene glycol Cu/ethylene glycol TiO <sub>2</sub> /water Al <sub>2</sub> O <sub>3</sub> /water
Duangthongs and Wongwises [109]	$\frac{k_{eff}}{k_f} = a + b\phi$ $a = 1.0225, b = 0.0272 \text{ for } T = 15^\circ\text{C}$ $a = 1.0204, b = 0.0249 \text{ for } T = 25^\circ\text{C}$ $a = 1.0139, b = 0.0250 \text{ for } T = 35^\circ\text{C}$	CuO/water TiO <sub>2</sub> /water TiO <sub>2</sub> /ethylene glycol TiO <sub>2</sub> /water nanofluid
Patel, et al. [48]	$\frac{k_{eff}}{k_f} = \left( 1 + 0.135 \left( \frac{k_p}{k_f} \right)^{0.273} \phi^{0.467} \left( \frac{T}{20} \right)^{0.547} \left( \frac{100}{d_p} \right)^{0.234} \right)$	Oxide and metallic nanofluids
Chandrasekar, et al. [110]	$\frac{k_{eff}}{k_f} = \left( \frac{C_{p,eff}}{C_{p,f}} \right)^a \left( \frac{\rho_{eff}}{\rho_f} \right)^b \left( \frac{M_f}{M_{eff}} \right)^c;$ $a = -0.023, b = 1.358, c = 0.125$	Al <sub>2</sub> O <sub>3</sub> /water nanofluids
Godson, et al. [133]	$\frac{k_{eff}}{k_f} = 0.929\phi + 0.9508$	EG/water nanofluids Ag/water nanofluids

**Table 3:** Models for thermal conductivity.

### Nanofluid Dynamical Models

Masuda observed the thermal enhancement due to the nanoparticle in 1993 and Choi tossed the term nanofluid in 1994. But it was not before 2006 that a mathematical model for the flow of nanofluids is described. Nanofluids have higher thermal conductivity and heat transfer coefficients than their base fluids. The change in viscosity and thermal conductivity is of the fluid effects its dynamics. Two mathematical models are developed to accurately describe the flow patterns and heat transfer effects are so-called Buongiorno's model and Tiwari-Das model.

**Buongiorno's model:** In 2006, Buongiorno [91] observed from the results of that these fluids shows an abnormal increase in thermal conductivity, viscosity and heat transfer rate coefficient. It is due to the nanofluid property of the Thermophoresis and temperature gradient. To validate they checked the relative motion of particles for inertial slip, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity. It emerges that only Brownian diffusion and thermophoresis affects the flow drastically.

To obtain the conservation equations of Nanofluids, consider the incompressible flow of nanofluid. It is assumed that both nanoparticle and base fluid is chemically inert hence no chemical reaction can take place. Also, it is assumed that the concentration of the particles is small. Nanoparticles and base fluid are assumed to be in equilibrium. Four equations describe the flows i.e. two continuity equations (for base fluid and for nanoparticles) and the single equation for both momentum and heat transfer.

The continuity equation for fluid can be

$$\nabla \cdot V = 0 \quad (11)$$

Here,  $V$  is the velocity of the fluid. For the particle, if  $J$  is diffusion mass flux of the nanoparticle w.r.t. fluid velocity, then it can be written as the sum of Brownian motion and thermophoresis if no external force is applied

$$-\frac{1}{\rho} \nabla \cdot J = \frac{\partial \phi}{\partial t} + V \cdot \nabla \phi = \nabla \cdot \left[ D_B \nabla \phi + D_B \frac{\nabla T}{T} \right] \quad (12)$$

Where  $\phi$  is the concentration of nanoparticles,  $D_b$  and  $D_T$  are due to slip velocity of particles cause by Brownian motion and thermoprosis. The momentum equation for nanofluids with non-external force or diffusion term is defined with as

$$\rho \left[ \frac{\partial V}{\partial t} + V \cdot \nabla V \right] = -\nabla P - \nabla \cdot \tau \quad (13)$$

$P$  is the pressure and  $\tau$  is the stress tensor.

The energy equations for nanofluids

$$\rho c \left[ \frac{\partial T}{\partial t} + V \cdot \nabla T \right] = -\nabla \cdot q + h_p \nabla \cdot J \quad (14)$$

Neglecting radiative heat transfer,  $q$  can be calculated as the sum of the conduction heat flux and the heat flux due to nanoparticle diffusion

$$q = -k \nabla T + h_p J \quad (15)$$

$c$  is heat capacity,  $h_p$  is specific enthalpy.

**Tiwari and Das' Model:** In this model [134], flow is assumed to laminar and incompressible. The model uses continuity, momentum and energy equations for a Newtonian fluid. If radiation heat transfer and other external forces are negligible and assuming constant thermal properties, the continuity, momentum and heat equation defined by the law of conservation of mass, momentum and energy respectively form the full flow model are defined as

For Continuity equation

$$\nabla \cdot V = 0 \quad (16)$$

For momentum equation

$$\rho_{nf} \left[ \frac{\partial V}{\partial t} + V \cdot \nabla V \right] = -\nabla P - \nabla \cdot \tau \quad (17)$$

The equation is traditional Navier-stokes equation. With  $\tau$  is defined usually for Newtonian base fluid as  $\tau = \mu_{nf} (\nabla \cdot V + \nabla \cdot V')$ .  $\mu_{nf}$  varies drastically with the concentration of nano particle (see Table. 2).

The energy equations for nanofluids

$$\rho_{nf} C_{nf} \left[ \frac{\partial T}{\partial t} + V \cdot \nabla T \right] = -\nabla \cdot q' \quad (18)$$

Neglecting radiative heat transfer,  $q'$  can be calculated as

$$Re q' = -k_{nf} \nabla T \quad (19)$$

### Conclusion

A number of essential conclusions and recommendations can be carried out regarding current knowledge and future research

1. Nanofluids shows a big revolution in recent decades that are fully elaborated in our literature. Many techniques were used in order to enhance the energy gain or loss for example in industry, in chemical reactors, in cooling of machine's engine, in refrigerator, in electrical, in solar energy, in electronic chips and also in heavy machineries

used in defense purpose.

2. Many significant results are introduced in recent decades about thermo-dynamical system. Reader can easily enhance their concepts of loss of energy in any system and then it is further used in various energy sectors.
3. Many advanced theoretical and experimental results that are discussed to help the reader to clarify their results with two dynamical models.
4. Models of thermal conductivity, Theoretical and experimental models of effective viscosity, Size and Shape effects of nanoparticles in nanofluids are increasing rapidly in current research work. Which helps the reader to easily use these models in their research.

### Nomenclature

Cu	Copper
Ag	Silver
Au	Gold
SiC	Silicon Carbide
SiN	Silicon Nitride
AlN	Aluminum Nitride
Al <sub>2</sub> O <sub>3</sub>	Aluminum Oxide
CuO	Copper Oxide
TiO <sub>3</sub>	Titanate
ZnO	Zinc Oxide
ZnS	Zinc Sulfide
HeC	Hydroxyethyl Cellulose
SiO <sub>2</sub>	Silicon Dioxide
EG	Ethylene Glycol
$\alpha$ , $\beta$ and $\sigma$	Semi-axes
X, y and z	Major axes
n	Empirical shape factor
$\psi$	Sphericity
nm	Nanometer
$\mu$	Viscosity
$\rho_f$	Fluid Density
$\varphi$	Nanoparticle volume fraction
$\rho_{nf}$	Nanofluid Density
$c_f$	Specific heat of fluid
$c_p$	Specific heat of particle

$c_{nf}$	Specific heat of nanofluid
$(\rho c)_{nf}$	Heat capacity of the nanofluid
$(\rho c)_f$	Heat capacity of the fluid
$(\rho c)_p$	Heat capacity of the nanoparticle
$\mu_{eff}$	Effective Viscosity
$\mu_f$	Fluid Viscosity
$\kappa_B$	Excess thermal-conductivity enhancement coefficient
$\rho_p$	Nanoparticle Density
$T, T_0$	Temperature
$\pi$	Pi
$C$	Specific Heat
A, B, C, a, b, c, n	Constants
M	Molecular weight of the base fluid
N	Avogadro number
$k_{eff}$	Effective Thermal Conductivity
$k_p$	Thermal conductivity of particle
$k_f$	Thermal conductivity of fluid
$k_s$	Thermal conductivity of solids
$D$	Einstein diffusion coefficient
Re	Reynolds Number
Pr	Prandtl Number
$T_{fr}$	Temperature of freezing point of the base liquid
$\lambda$	Water molecules mean free path
$\phi_p$	Total particle volume fraction
$V$	Velocity
$\phi_f$	Total volume fraction



P	Pressure
$\tau$	Newtonian Base fluid
q	Energy flux
$h_p$	Nanoparticle specific enthalpy
J	Mass flux

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