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Logarithm Model for Decline Curve Analysis

Fanchi JR*

Department of Engineering and Energy Institute, Texas Christian University, USA

***Corresponding author:** John R Fanchi, Department of Engineering and Energy Institute, Texas Christian University, USA, Email: j.r.fanchi@tcu.edu

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Abstract

A decline curve analysis model has been developed for flow rate proportional to the logarithm of time. The logarithm of time model requires careful interpretation of initial time dependence of volumetric flow rate. Applications of the model to shale gas production and shale oil production are presented.

Introduction

A typical decline curve analysis (DCA) model expresses flow rate q as a function of time $f(t)$ so that $q = f(t)$ with a first order time derivative $\frac{dq}{dt} = \frac{df(t)}{dt}$. Arps JJ [1] decline curves are solutions of the equation

$$\frac{dq}{dt} = -aq^{n+1} \quad (1)$$

The factors a and n are empirically determined and are constant with respect to time t. The empirical constant n ranges from 0 to 1. The shape of the decline curve depends on the value of n as shown in Table 1. The term q_i is initial flow rate.

Decline Curve	n	$q = f(t)$
Exponential	0	$q = q_i e^{-at}$
Hyperbolic	$0 < n < 1$	$q = \sqrt[n]{1/\left(nat + q_i^{-n}\right)}$
Parabolic	1	$q = 1/\left(nat + q_i^{-1}\right)$

Table 1: Arps JJ [1] Decline Curves.

Decline curve analysis provides information which can be used to estimate cumulative production N. If we assume a functional relationship $q = f(t)$ is known, cumulative production is the integral over flow rate q for a period of time that ranges from initial time t_0 to final time t, thus

$$N = \int_{t_0}^{t} q\,dt = \int_{t_0}^{t} f(t)\,dt \quad (2)$$

As an example, cumulative production N for Arps exponential decline equation is

$$N = \int_0^t q\,dt = \frac{q_i - q}{a} \quad (3)$$

where flow rate is integrated over time from initial rate q_i at initial time $t_0 = 0$ *to* rate q at time t. Time periods such as day, month and year need to be consistently applied. For more discussion of Arps equation and decline curve analysis, see Lee [2], Sun [3], Fanchi and Christiansen [4], Ezekwe [5], Panja and Wood [6].

Another way to infer the time dependence of flow rate is to consider different types of flow. Flow regime models provide different relationships between pressure drop and time. As a first approximation, well flow rate during primary depletion is proportional to pressure drop Δp where pressure drop is the difference in pressure $\Delta p = p_i - p_{wf}$ between initial reservoir pressure p_i and wellbore flowing pressure p_{wf}. The relationship between flow rate and time can be obtained by writing productivity index $J = q/\Delta p$

as $q = J \times \Delta p$ and replacing pressure drop with its time-dependent form for different flow models. Table 2 shows relationships between flow rate q and time t for some flow regime examples.

$q = f(t)$	Constants	Flow Regime Examples
$q = at^n + b$	*a, b, n*	Rate q proportional to t^n Volumetric flow (n = 1) Linear flow (n = ½) Bilinear flow (n = ¼)
$q = a\ln t^n + b$	*a, b, n*	Rate q as function of ln t^n Radial flow (n = 1)

Table 2: Rate *q* as a Function of Time *f(t)* for Different Flow Regimes.

The purpose of this paper is to develop a decline curve analysis model for flow rate proportional to logarithm of time *ln t*. This model is referred to here as the logarithm model (LNDM).

Logarithm Model (LNDM)

The radial flow relationship between flow rate and time in Table 2 suggests that flow rate *q* is proportional to the logarithm of time *ln t* in the form

$$q = a\ln t + b \quad (4)$$

where *a, b* are constant with respect to time and n = 1. The differential equation for the rate-time dependence in Equation (4) is obtained by calculating the time derivative

$$\frac{dq}{dt} = \frac{d\left(a\ln t + b\right)}{dt} = \frac{a}{t} \quad (5)$$

Equation (5) is the differential equation for the logarithm model LNDM and is the analog of Equation (1). The general solution of Equation (5) is

$$q = a\ln\ t + b \text{ where } b = \text{-a ln } \lambda \quad (6)$$

We have introduced the parameter λ to show how to make physical units consistent. To demonstrate this, substitute $b =$ -a ln λ into Equation (6) to obtain

$$q = \text{ a ln t} - \text{a ln } \lambda = a\ln\left(t/\lambda\right) \quad (7)$$

The units are consistent when λ has the unit of time so that t/λ is dimensionless. The slope *a* has the same unit as flow rate *q* and $a < 0$ for a flow rate that declines as time increases. The parameter λ is expressed in terms of *a, b* by $\lambda = \exp\left(-b/a\right)$.

Cumulative Production and Economic Limit

Cumulative production for the logarithm model is given by the integral

$$N = \int_{\tau_0}^{\tau}\left(a\ln\ t + b\right)dt \quad (8)$$

for the time interval $\tau_0 \le t \le \tau$. We solve the integral in Equation (8) using $\int \ln\ axdx = x\ln ax - x$ to find

$$N = a\left[t\ln\ t - t\right]\Big|_{\tau_0}^{\tau} + bt\Big|_{\tau_0}^{\tau} \quad (9)$$

or

$$N = a\left(\tau\ln\ \tau - \tau\right) - a\left(\tau_0\ln\ \tau_0 - \tau_0\right) + b\left(\tau - \tau_0\right) \quad (10)$$

The maximum production time occurs when flow rate $q \to 0$ since flow rate $q < 0$ when $t > \tau$. We have the condition

$$q\left(t = \tau\right) = 0 = a\ln\ \tau + b \quad (11)$$

Solving for τ gives

$$\tau = \exp\left(-\frac{b}{a}\right) \quad (12)$$

The economic limit occurs at time $\tau_{econ} \le \tau$.

An alternative economic limit can be set for a non-zero economic rate q_{econ} using the following procedure. The time corresponding to the non-zero economic rate q_{econ} is found using the equation

$$q\left(t = t_{econ}\right) = q_{econ} = a\ln\ t_{econ} + b \quad (13)$$

The time corresponding to the economic limit t_{econ} is then

$$t_{econ} = \exp\left[\left(q_{econ} - b\right)/a\right] \quad (14)$$

Initial Condition of LNDM Model

The logarithm model LNDM requires a careful interpretation of volumetric flow rate as a function of time when t goes to 0. The usual initial condition for models such as the familiar Arps models in Table 1 uses flow rate $q(t)$ at the beginning of the time period, that is flow rate at $q(t = 0)$. In the case of the LNDM model, we cannot use $\ln(q(t))$ at time $t = 0$.

An alternative approach that is suitable for the LNDM model is to recognize that volumetric flow rate data are typically reported as the average volume of fluid produced in a period of time. For example, the first volumetric flow rate may be the volume produced in a day, a month, or a year. If we choose to view the flow rate as the volume produced throughout the time period rather than an instantaneous rate at the beginning of the time period, then we can define an integral that begins at the completion of time period 1. In practice, we can use daily, monthly, or annual rates of production versus time and integrate over time beginning at the completion of the first time period rather than at the start of the first time period. Initial production rate, or IP, is the volume produced during the first time period divided by the duration of the time period.

Application of the LNDM Model using Linear Regression Analysis

The logarithm model LNDM is applied by first rewriting the model $q = a \ln t + b$ as a straight line $y = \alpha x + \beta$ where α is the slope and β is the intercept of the straight line. The relationships between models are

$$y = q, x = \ln t, \alpha = a, \text{ and } \beta = b \quad (15)$$

Linear regression analysis is used to fit rate versus time data to the straight-line model $y = \alpha x + \beta$. Logarithm model LNDM parameters α, b, λ are then obtained from linear regression parameters α, β using Equation(15), namely $a = \alpha$, $b = \beta$, and $\lambda = \exp(b/a)$.

Example: LNDM Model of Shale Gas Decline

The LNDM model was used to forecast shale gas recovery for four major North American shale gas fields [7]: Barnett, Fayetteville, Haynesville, and Woodford. As an example, the LNDM parameters for a high flow rate gas well are a = -1.0 x 10^4 and b = 1.5 x 10^5. The LNDM model parameters were calculated using monthly gas production so time is expressed in months. The gas rate after 3 years of production is calculated at t = 36 months so that

$$\begin{aligned} q = a \ln(t) + b &= -1.0 \times 10^4 \times \ln(36) \\ &+ 1.5 \times 10^5 = 1.14 \times 10^5\, MSCF/mo \end{aligned} \quad (16)$$

Application of the Logarithm Model to Shale Gas Production

The logarithm model LNDM was applied to four major North American shale gas fields: Barnett, Fayetteville, Haynesville, and Woodford [7]. Figures 1 and 2 present the regression fit of the LNDM model to shale gas production type curve for the Barnett and Woodford shales respectively [8]. The LNDM model parameters from linear regression are given in Table 3. Rates expressed in Mscf/d define the units of a and λ as Mscf/d and day respectively.

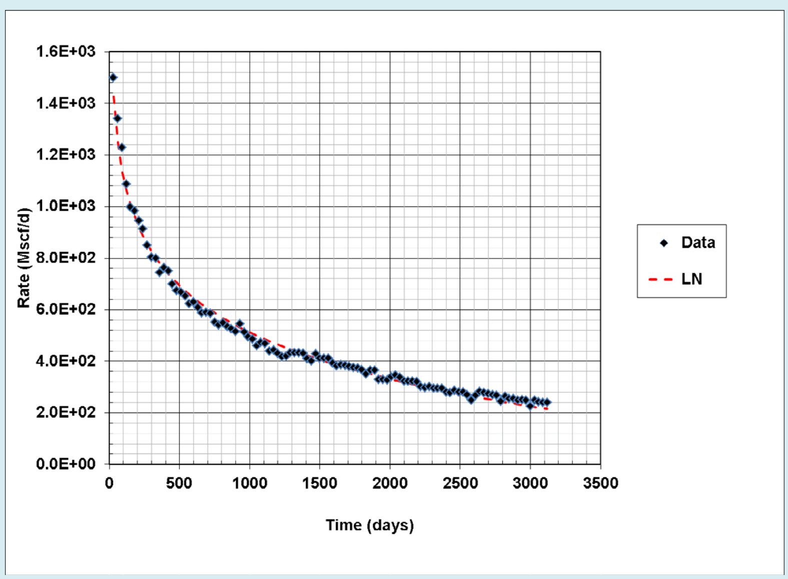

Figure 1: Regression fit of the LNDM model to Barnett shale gas production.

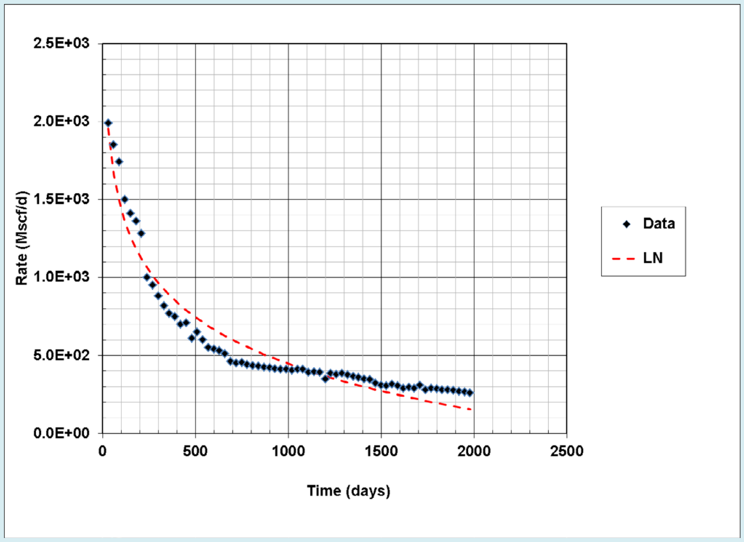

Figure 2: Regression fit of the LNDM model to Woodford shale gas production.

Shale	Constant	Flow Regime Examples
Barnett	a b λ	-261.1 2315.8 0.000141
Woodford	a b λ	-430.17 3419.0 0.000353

Table 3: LNDM Parameters for Shale Gas Examples.

The fit of the LNDM model to the Barnett shale gas production type curve is a reasonable fit, while the fit of the LNDM model to the Woodford shale gas production type curve is not as good a fit. It is reasonable to expect that the LNDM model may not be suitable for all applications.

Application of the Logarithm Model to Shale Oil Production

Figures 3 and 4 present the regression fit of the LNDM model to shale oil production data for the Eagle Ford and Bakken shales respectively [8]. Applications of the hyperbolic decline curve model and a stretched exponential decline curve model to shale oil production data are illustrated in Fanchi [9]. The LNDM model parameters from linear regression are given in Table 4. Rates expressed in Bbl/d define the units of a and λ as Bbl/d and day respectively.

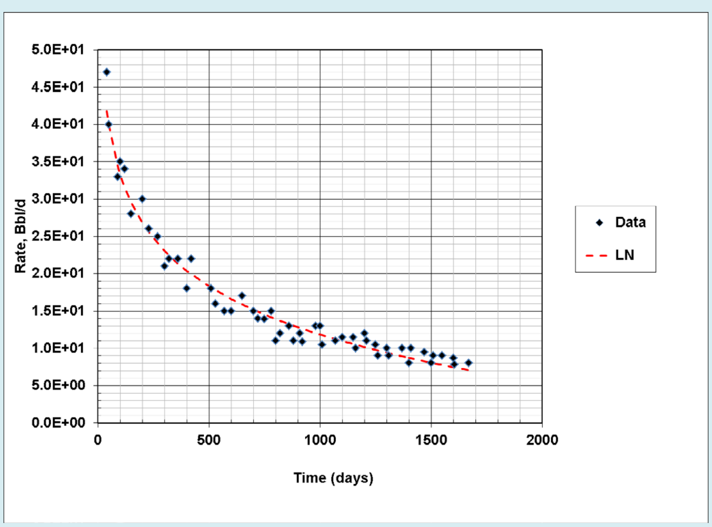

Figure 3: Regression fit of the LNDM model to Eagle Ford shale oil production.

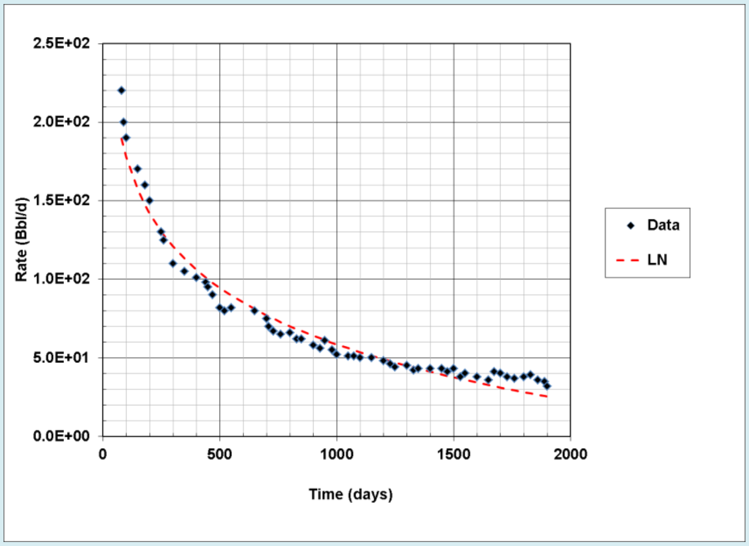

Figure 4: Regression fit of the LNDM model to Bakken shale oil production.

Shale	Constant	Flow Regime Examples
Bakken	a	-51.785
	b	416.32
	λ	0.000323
Eagle Ford	a	-9.3114
	b	76.185
	λ	0.000280

Table 4: LNDM Parameters for Shale Oil Examples.

The fit of the LNDM model to the Eagle Ford shale oil production type curve is reasonable. The fit of the Bakken shale oil production type curve may be considered a reasonable representation, but another decline curve model might provide a better fit of the nonlinearity of the type curve.

Conclusions

A decline curve analysis model called the logarithm model (LNDM) was developed for flow rate proportional to the logarithm of time. The functional relationship between flow rate and logarithm of time was inferred from a study of flow regimes. We show how to use the rate-time relationship to calculate cumulative production at a specified economic limit for the LNDM model.

The use of the logarithm of time requires a careful interpretation of volumetric flow rate at the initial condition. The initial condition for the LNDM model states that the initial flow rate is the volume of production during the first period of time divided by the first period of time.

The LNDM model was illustrated by applying it to shale gas production and shale oil production decline curves. The decline curves represent a selection of North American shales.

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