MEDWIN PUBLISHERS
Committed to create value for Researchers

Psychology & Psychological Research International Journal

Exploring Metacognitive Processes Involved in Solving Mathematical Problems

Trinidad G[1], Debora A[1], Steve L[2] and Celestino R[1,*]

[1]Faculty of Psychology, University of Oviedo, Spain

[2]Faculty of Psychology, University of Newcastle-Australia

Review Article

Volume 2 Issue 3

Received Date: July 31, 2017

Published Date: August 05, 2017

***Corresponding author:** Celestino Rodriguez Perez, Faculty of Psychology, University of Oviedo, Plaza Feijoo s/n, 33003 Oviedo Asturias, Room 225, Spain, Phone: +34609939277; E-mail: rodriguezcelestino@uniovi.es

Abstract

Mathematical problem solving is a complex activity which depends upon multiple factors for correct performance. In this context, much scientific evidence suggested that, to be successful, students need to be self-regulated in their problem-solving activities, thereby exercising effective control over their cognitive, affective and behavioral processes. This mini-review briefly shows (1) a method for evaluating the processes involved in solving mathematical problems, (2) the explanatory potential of metacognitive processes in judgments-accuracy after task completion, and (3) the effects of other cognitive, affective, motivational and behavioral variables on these judgments.

The results showed the usefulness of an evaluation method that has been designed as a measure of the processes involved in solving mathematical problems, and also the need to consider a wider range of variables when differences in calibration cannot otherwise be explained.

Keywords: Problem solving; Metacognition; Assessment; Judgments

Introduction

Problem solving has always been regarded as a key point of mathematics, which explains the amount of research conducted on this issue. It is a complex activity which depends upon multiple factors for correct performance. Because of its relevance enhancing the development of basic skills such as analysis, comprehension, reasoning and application, problem solving, is considered a core competence individuals need to acquire for today's world. Nevertheless, previous studies have shown that students at different educational stages usually struggle with mathematics problems while recognizing that the attempts to teach students general problem solving strategies have been mostly unsuccessful [1,2]. The results of the Programme for International Students Assessment (PISA) of 2014 [3] and previous years have highlighted this reality.

Obviously, there is a wide range of variables that may explain these results. Thus, addressing all of them is simply not feasible. Among the possible variables related to successful mathematical problem solving, strategy use based on Self-Regulated Learning (SRL) has demonstrated to be an important determining factor [4-7]. These studies emphasize the relevance of the cognitive, affective and motivational processes involved in solving mathematical problems and the control that students exercise over them. However, focusing on the cognitive sphere, previous research suggests that students tend to show poor metacognitive skills while involved in mathematical problem-solving situations. This has a negative impact not only on task performance but also on student attitudes, motivation, and perceptions or judgments of performance [6,8,9].

An important metacognitive mechanism related to these judgments is the so-called *"calibration",* referred to as the degree to which one´s judgments of performance corresponds to one´s actual performance, the latter determined on the basis of an objective measure such as a score on a test [10]. These judgments may be made before –"predictions"– or after –"postdictions"– task completion, and their accuracy has demonstrated to be an important predictor of mathematics achievement and problem-solving performance. However, students have demonstrated to be poorly calibrated in previous studies, showing a tendency towards over-confidence, which has adverse effects on students´ effort, persistence and interests towards a subject or task [11-13].

Although both components –predictions and postdictions– are understood as important parts of calibration, the latter –also called post-performance judgments– would be more accurate and reliable as they inform about monitoring mechanisms during task performance [14]. Therefore, this sort of judgments can be examined from the perspective of the metacognitive process shown by students while solving mathematical problems. Although the origin of these judgments has been substantially studied, the research conducted on this issue presents some limitations: first, the number of references regarding calibration in Elementary School stages is rather scarce, as most of the studies have been carried out with college or undergraduate samples; second, the potential explanatory power of the process over this kind of judgments has not been analyzed to date; finally, just a few studies have addressed the analysis of the determining factors of calibration from a comprehensive perspective, focusing on analyzing a small range of variables instead.

According from previous literature and within this context, it is important to answer the following questions:

1. How can the process be assessed, and what does this process reveal about the metacognitive mechanisms involved in solving mathematical problems?
2. Can the process provide information about the mechanisms involved in making post-performance judgments in these tasks?
3. Are there other affective, motivational, cognitive or behavioural variables that may be explaining differences in these judgments?

Assessment Process and the Metacognitive Mechanisms Involved in Solving Mathematical Problems

The Triple Task Procedure in Mathematics –*TTPM*– examines its usefulness in the assessment of the metacognitive processes involved in mathematical problem solving. This measure is an adaptation of the Triple Task technique [15-17], initially proposed to study the processes involved in composition writing. This is one of the so-called "on-line" assessment tools [17] referred to as those measures taken concurrently with performance. This new version of the Triple Task is based on the Self-Regulation model of Zimmerman [18] and the IDEAL problem-solving model of Bransford and Stein [19].

Results from García et al. [20], evidenced the existence of a great variability in the process showed by students, as the high standard deviations found suggested. This process was characterized by the application of ineffective planning strategies, a preference to use familiar procedures, such as doing calculations, and a lack of evaluation mechanisms in the general simple. The analysis of the differences between groups with different performance in the problems (Success vs. Failure) revealed however the sub-process involved in planning, mainly the use of information representation and organization strategies, as important determining factor of students´ success.

Process Information about Problems Solving Task and the Mechanisms Involved in Making Post-Performance Judgments

Other study carried out by Garcia et al [21] established an estimation of post-performance calibration accuracy (how accurate are them in their judgments? Did they show a tendency towards over- or under- confidence?) and the stability of both post-performance judgments and actual performance was analyzed. Then, differences in the metacognitive process between students with different calibration accuracy (Accurate vs. Inaccurate) were examined. The results indicated the presence of low levels of calibration accuracy among students, showing a strong tendency towards over-confidence. Additionally, students showed a high stability in their judgment and actual performance. As for the differences in the metacognitive process, inaccurate students used strategies of information representation and organization to a lesser extent than their peers in the accurate group, showing resolution patterns coherent with a use of "trial and error" mechanisms instead. Finally, different levels in mathematics achievement led to different calibration patterns. Specifically, judgments accuracy progressively improved as mathematics achievement level increased. Differences in the metacognitive process itself were also found, mainly regarding the use of information representation and organization strategies, being confirmed a positive relationship between them. Grade level did not generate a specific pattern of differences in this sense.

Affective, Motivational, Cognitive or Behavioural Variables Explained Differences in Post-Performance Judgments

Given the characterization of calibration as an important metacognitive mechanism, it is important to examine the potential usefulness of executive functions and a deep approach to learning predicting these judgments, starting from the analysis of the relationship between these variables and metacognition (components of knowledge and skills, the latter defined as application of this knowledge in learning situations). Furthermore, it is also relevant to analyse the extent to which the variables that showed to be significant in these previous studies, along with affective-motivational variables related to mathematics, achievement in the subject, and task characteristics, may predict differences in post-performance calibration.

While studies linking Metacognition and executive functioning are practically non-existent to date, the characterization of executive functions as high-order control mechanisms suggests that both executive functions and metacognition should keep some kind of association [17]. If this association were found, executive functions would therefore be a good candidate to explain some of the variability in the judgments made by students, especially considering the important relationship between executive functioning and performance in mathematics, evidenced in the literature.

A recent research examined the relationship between executive functions and metacognition focusing on metacognitive knowledge as it is suggested to be the basis for metacognitive skills development [23]. In this sense, this study starts by analyzing if students with different metacognitive knowledge levels (Low vs. High) differed in their metacognitive skills, conceptualized as the application of this knowledge in general learning situations, specifically in the Self-Regulated Learning (SRL) phases of planning, execution and evaluation.

These two components were analyzed by means of a strategy recognition test –*Learning Strategies Knowledge Questionnaire-LSKQ– [25,26]* and self-report –SRL Processes Inventory-SRLPI–[27], respectively. The results showed that students with high metacognitive knowledge reported using metacognitive strategies, such as thinking about the steps or materials needed to perform a task, comparing their final result in a task with the result they expected to obtain at the beginning, or keeping track of their own progress during the task, more frequently than their peers with low metacognitive knowledge. These differences were statistically significant in the Planning and Execution phases of SRL measured with *SRLPI* [27]. Regarding executive functions, statistically significant differences between groups in the components of sustained attention, focus, working memory and planning were found. These results were consistent using both scales of executive functions. Families and teachers reported enhanced levels of executive functioning in the group of students with high metacognitive knowledge.

Based on previous studies on the subject, which suggest the existence of an association between the use of a deep learning approach and better metacognitive strategies in problem solving, this study aimed at analyzing the relationship between this approach and the components of metacognitive knowledge and skills. The results of Garcia, et al. [23] indicated the existence of statistically significant differences between groups in metacognitive knowledge. In this regard, the increased use of a deep approach to learning was associated with better overall metacognitive knowledge. A clear pattern of differences in metacognitive skills during problem-solving tasks was not found, although minimal differences were found in the second mathematical problem, mainly in the sub-process of revision. Specifically, a high use of a deep approach was associated with increased use of revision strategies. These results are discussed from the point of view of the role of metacognitive knowledge as a basis for the development of metacognitive skills, and the distinction between “off-line” vs. “on-line” assessment measures; defined the former as those measures taken after of before task performance –such as questionnaires–, and the latter as measures taken concurrent with task performance –Triple Task Procedure in Mathematics in this case.

Finally, the predictive value that executive functions, and other variables that literature revealed as having influence on calibration, may have over post-performance judgments accuracy is examined in another recent study of Garcia, et al. [24].

This work analyzed the extent to which the following variables predicted post-performance judgments accuracy: executive functions, affective-motivational components related to mathematics, achievement in this subject and task characteristics (perceived difficulty and total time spent on the problem-solving tasks). Prior to analysing their predictive value, differences in these variables between groups with different calibration (Accurate vs. Inaccurate) were examined. Students were assigned to these two groups based on the accuracy of their post-performance judgment in both mathematical problems. Students who judge accurately their performance only in one of the problems were excluded from the analyses to avoid hazard responses. Thus, a sample of 188 students was used in this study. Executive functions were assessed by means of the EFE scale Executive Functioning Scale for Teachers described above, only in its form to families, in order not to lose excessive sample size[28].

Regarding differences between groups with different calibration accuracy, the results indicated that accurate students showed improved motivations, beliefs and attitudes towards mathematics, as well as higher levels of achievement in the subject than their peers in the inaccurate group. Accurate students were evaluated by families as significantly less impulsive, also reporting better attention, memory, planning and organization skills in this group. They also spend more time solving problems than the inaccurate group, mainly in the first problem. Finally, with regard to the predictive value of these variables, mathematics performance, perceived usefulness or value of the subject and the total time spent solving the first problem, significantly predicted differences in calibration, classifying 71.3% of the sample correctly.

The results obtained in these five studies showed the usefulness of the designed measure –Tripe Task Procedure in Mathematics (TTPM) – in the evaluation of the metacognitive process involved in solving mathematical problems and the potential value of this process explaining differences in post-performance calibration. Calibration is, however, a complex mechanism which depends upon multiple variables, being necessary to address this analysis from a comprehensive perspective.

Conclusions

The method of assessing the process that was designed and tested (the Triple Task Procedure in Mathematics) was found to be useful in the study of the metacognitive processes involved in solving mathematical problems. This method provides evidence about the profile of the process shown by students, enabling the establishment of a relationship between process and product (or task-result), as well as between process and post-performance calibration [20].

The application of this assessment method helps to identify certain strengths and weaknesses in student problem-solving activities that are especially relevant from the perspective of *Teaching-Learning* processes. Both teachers and students themselves could benefit from this type of information by utilising it to attain improvements in this area, which in turn, would also allow students to adopt increasing degrees of control over their own learning processes.

On the other hand, it is essential to take into consideration a student’s individual characteristics in order to fully promote improvements in their ability to solve mathematical problems. The great variability shown by students while solving mathematical problems suggests that there is a need to pay attention to the individual characteristics observed during the process [21-23].

Additionally, differences in mathematics achievement are also an important variable to consider in this context. Different achievement levels may be related to differential patterns of problem solving and even to differences in the accuracy of performance judgments made by students [24]. Previous research has shown that an important part of the effectiveness of interventions aimed at improving problem-solving skills and calibration depends largely upon this variable. Moreover, it is important to pay special attention to students with low levels of academic achievement in this area by addressing different variables in the analysis, from the characteristics of student problem-solving processes to their motives as well as their preferences for the use of certain procedures while performing the task.

Regarding to the calibration process, it involves a complex process that relies on multiple factors, and thus must be examined from a comprehensive analysis perspective [10]. It is necessary to study a wide range of variables (working memory, inhibition, impulsivity, attention, etc.) in order to obtain a complete picture of the factors that influence the formation and accuracy of judgments. These factors must also correspond to different levels of analysis, from general aspects such as cognitive abilities or behavioral control, to more specific such as the characteristics of the task.

For this reason, it is also important to note how different variables (even those restricted to different levels) interact with one another. A better understanding of these factors and their interrelationships will allow us to design intervention strategies that are potentially more effective [24,25].

Finally, certain aspects of the assessment procedure and the research design must be taken into consideration before the results obtained in the different studies included in this work can be fully interpreted and applied to future research on this issue. For example, it is possible that the type of assessment measure used, taken concurrently with the completion of the task and based on directed introspection, has an effect on the process itself (i.e., resulting reactive).

References

1. Blanco LJ, Guerrero E, Caballero A (2013) Cognition and affect in Mathematics problem solving with prospective teachers. The Mathematics Enthusiast 10(1-2): 335-364.

2. Santos M (2007) Mathematical problem solving. Cognitive foundations. México: Trillas.

3. OECD (2003) Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills. PISA: OECD Publishing.

4. Lazakidou G, Retalis S (2010) Using computer supported collaborative learning strategies for helping students acquire self-regulated problem-solving skills in mathematics. Comput Educ 54: 3-13.

5. Montague M (2008) Self-Regulation and Mathematics Instruction. Learn Disabi L Res Pr 22(1): 75-83.

6. Pennequin V, Sorel O, Nanty I, Fontaine R (2010) Metacognition and low achievement in mathematics: The effect of training in the use of metacognitive skills to solve mathematical word problems. Think Reasoning 16(3): 198-220.

7. Schmitz B, Perel P (2011) Self-monitoring of self-regulation during math homework behaviour using standardized diaries. Metacogn Learn 6(3): 255-273.

8. Montague M, Enders G, Dietz S (2011) Effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities. Learn Disability Q 34(4): 262-272.

9. Pereis F, Dignath C, Schmitz B (2009) Is it possible to improve mathematical achievement by means of self-regulation strategies? Evaluation of an intervention in regular math classes. Eur J Psychol Educ 24(1): I7-29.

10. Hacker DJ, Bol L, Keener MC (2008) Metacognition in education: A focus on calibration. In Dunlosky J and Bjork R (Eds.). Handbook of Memory and Metacognition. Mahwah, NJ: Erlbaum, pp. 429-455.

11. Jacobse AE, Harskamp EG (2012) Towards efficient measurement of metacognition in mathematical problem solving. Metacogn Learn 7(2): 133-149.

12. Ozsoy G (2012) Investigation of fifth grade students" mathematical calibration skills. Educ Sci-TheorPract 12(2): 1190-1194.

13. Rinne LF, Mazzocco MM (2014) Knowing right from wrong in mental arithmetic judgments: Calibration of confidence predicts the development of accuracy. Plos One 9(7).

14. McCormick CB (2003) Metacognition and Learning. In Reynolds WB and Miller GE (Eds.). Handbook of psychology: Educational psychology. Hoboken, NJ: Wiley pp: 79-102.

15. Kellog RT (1987) Writing performance: Effects of cognitive strategies. Writ Commun 4: 269-298.

16. Olive T, Piolat A (2002) Suppressing visual feedback in written composition: Effects on processing demands and coordination of the writing processes. Int J Psychol 37(4): 209-218.

17. Veenman MVJ (2011) Learning to self-monitor and self-regulate. In Mayer R and Alexander P (Eds.), Handbook of research on learning and instruction. New York, NY: Rout ledge pp: 197-218.

18. Zimmerman B (2000) Attaining self-regulation. A social cognitive perspective. In Boekaerts M, Pintrich PR and Zeidner M (Eds.). Handbook of self-regulation. San Diego, CA: Academic Press, pp: 13-39.

19. Bransford JD, Stein BS (1993) The ideal problem solver: A guide for improving thinking, learning and creativity (2nd Ed.). New York, NY: WH Freeman.

20. Garcia T, Betts L, Gonzalez-Castro P, Gonzalez-Pienda JA, Rodriguez C (2016) On-line assessment of the process involved in Maths problem-solving in fifth and sixth grade students: Self-regulation and achievement. Rev Lat Am Inv Mat E 19(2): 165-185.

21. Garcia T, Rodriguez C, Gonzalez-Castro, P, Gonzalez-Pienda JA, Torrance M (2016) Elementary students' metacognitive processes and post-performance calibration on mathematical problem-solving tasks. Metacogn Learn 11(2): 139-170.

22. García T, Rodríguez C, González-Castro P, Álvarez-García D, González-Pienda JA (2016) Metacognition and executive functioning in Elementary School. An Psicol 32(2): 474-483.

23. García T, Cueli M, Rodríguez C, Krawec J, González-Castro P (2015) Metacognitive Knowledge and Skills in Students with Deep Approach to Learning. Evidence from Mathematical Problem Solving. Rev Psicodidact 20(2): 209-226.

24. García, TG, Kroesbergen E, Pérez CR., González-Castro P, Gonzalez-Pienda JA (2015) Factors involved in making post-performance judgments in mathematics problem-solving. Psicothema 27(4): 374-380.

25. Núñez JC, Cerezo R, Bernardo A, Rosário P, Valle A et al. (2011) Implementation of training programs in self-regulated learning strategies in moodle format: Results of an experience in higher education. Psicothema 23(2): 274-281.

26. Rosário P, Mourão R, Núñez JC, González-Pienda JA, Solano P (2006) School-family: Is a reciprocal and positive relationship possible? Papelesdel Psicólogo 27(3): 171-179.

27. Rosário P, José Carlos, Núñez Pere J. Ferrando, Maria Olímpia Paiva, Abílio Lourenço, et al. (2013) The relationship between approaches to teaching and approaches to studying: A two-level structural equation model for biology achievement in high school. Metacogn Learn 8(1): 44-77.

28. García T, Álvarez-García D, González-Castro P, Álvarez L, Segurola LA (2014) Psychometric properties of the Executive Functioning Scale for Teachers. Rev Psic y Educ 9(1): 49-69.