

Appendix

Probability of EPR-Generated Hydrogen Bond Arrangements, *keto-amino* → *enol-imine*, Using Approximate Quantum Methods

Natural selection has designed duplex genomes for the time-dependent populating of initially unoccupied, but energetically accessible, enol and imine proton qubit states as consequences of quantum uncertainty limits, $\Delta x \Delta p_x \geq \hbar/2$, operating on metastable amino ($-\text{NH}_2$) hydrogen bonding protons [32-37,42-44]. The resulting quantum confinement introduces direct quantum mechanical proton – proton physical interaction in too small of space, Δx , thereby generating probabilities of EPR arrangements, *keto-amino* → *enol-imine*, which create position and momentum entanglement between separating product protons [10-12,16-19]. Each reduced energy product proton is shared between two indistinguishable sets of electron lone-pairs belonging to enol oxygen and imine nitrogen, and thus, participates in entangled quantum oscillations at $\sim 10^{13} \text{ s}^{-1}$ between near symmetric energy wells within *intramolecular* decoherence-free subspaces, until “measured by” Grover’s-type [20] enzyme quantum processors [16-19,23-25,44-45]. In intervals $\delta t \ll 10^{-13} \text{ s}$, the quantum reader “traps” an entangled oscillating qubit, H^+ , in a DNA groove [38]. This creates an enzyme – proton entanglement that instantaneously specifies explicit instructions for an entangled enzyme quantum search, $\Delta t' \leq 10^{-14} \text{ s}$, to select the correct incoming tautomer for pairing with the ultimately decohered eigenstate, which forms the observable molecular clock substitution, **ts** [16,31,35-36,42-44]. Since specification of the molecular clock, **ts** or **td**, is completed before proton decoherence, $\Delta t' < \tau_D < 10^{-13} \text{ s}$, a feedback loop exists between an entangled enzyme quantum processor “measurement” and initiation of duplex genome evolution [16-19].

For purposes of discussing metastable keto-amino states populating reduced energy enol and imine, dynamic entangled proton qubit states, $\text{G-C} \rightarrow \text{G}'\text{-C}'$ and $\text{G-C} \rightarrow \text{*G}\text{-*C}$, time-dependence for the reactive five proton system of metastable G-C to populate complementary entangled proton qubit states is modeled in terms of a “composite” proton, in an asymmetric three-well potential, illustrated in Figure 11. An expression is obtained for the quantum mechanical “rate constant”

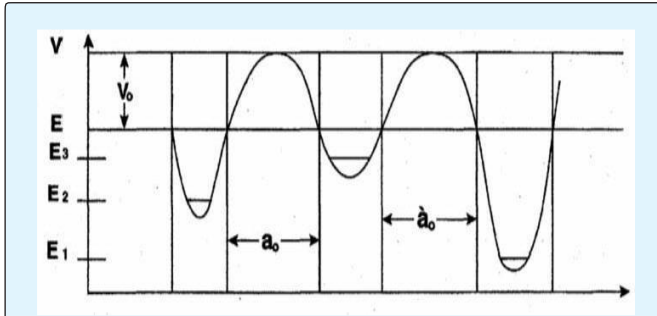
associated with the EPR hydrogen bond arrangement, *keto-amino* → *enol-imine*, via symmetric and asymmetric channels (Figures 2&3; Tables 1 & 2). This allows development of a polynomial expression for an evolving Darwinian, genomic system to express time-dependent alterations – classical + EPR-generated entanglements – in genetic specificities at DNA base pairs within a particular gene [16-19,39,42-44]. Here the motion of two tunneling-exchange protons, using the symmetric and asymmetric channels (Figures 2&3), is simulated in terms of a composite proton model. Secondary contributions by the 2nd asymmetric pathway (unlabeled) are neglected. At $t = 0$, the composite proton is replicated into the metastable state $|3\rangle$ at energy E_3 which, per data and shown in Figure 11, is separated from the enol-imine ground state, $|1\rangle$, and hybrid state, $|2\rangle$, by approximately equal energy barriers. The relationship $E_1 < E_2 < E_3$ for the ground state, hybrid state and metastable state, respectively, is displayed in Figure 11. Enol-imine product states are designated by a general arrangement state, $|\rho\rangle$, where the energy E_ρ would equal E_1 or E_2 as appropriate. Time-dependence of an eigenstate, $|\Psi\rangle$, is expressed by $|\Psi\rangle = |\varphi_1\rangle \exp(-i E_1 t / \hbar)$, so $|\Psi\rangle = |\varphi_1\rangle$ at $t = 0$ [33,118-119]. The relationship $|\Psi\rangle = \sum_i |i\rangle \langle i|\Psi\rangle$ is used to express an eigenstate $|\Psi\rangle$ in terms of base states $|i\rangle$ and amplitudes C_i as

$$|\Psi\rangle = |1\rangle \langle 1|\Psi\rangle + |2\rangle \langle 2|\Psi\rangle = |1\rangle C_1 + |2\rangle C_2 \quad (23)$$

where base states satisfy $\langle i|j\rangle = \delta_{ij}$. The eigenstate is normalized, $\langle \Psi|\Psi\rangle = 1$, and an eigenstate and eigenvalue E are related to the Hamiltonian matrix, $\Sigma_{ij} \langle i|H|j\rangle$, by $\Sigma_j \langle i|H|j\rangle \langle j|\Psi\rangle = E \langle i|\Psi\rangle$, which can be rewritten as

$$\Sigma_j (H_{ij} - E \delta_{ij}) C_j = 0 \quad (24)$$

for an expression to solve for amplitudes, $\{C_k | i=1,2; j=1,2\}$. A nonzero solution to Equation (24) is available if the determinant of $\Sigma_j (H_{ij} - E \delta_{ij}) = 0$.



(Figure 11: Asymmetric three-well potential to simulate metastable keto-amino protons populating accessible enol-imine states in terms of a “composite” proton originating in the metastable E_3 energy well at $t = 0$ where $E_1 < E_2 < E_3$).

A two-level Hamiltonian that will allow a composite proton to tunnel from the metastable state $|3\rangle$ at energy E_3 , to an arrangement state $|\rho\rangle$ at energy E_ρ , can be written as

$$H = \begin{pmatrix} E_3 & -\alpha_\rho \\ -\alpha_\rho & E_\rho \end{pmatrix} \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \quad (25)$$

where α_ρ is the quantum mechanical coupling between states $|3\rangle$ and $|\rho\rangle$. The resulting upper and lower eigenvalues, $E_{A\rho}$ and $E_{B\rho}$, are found as

$$E_{A\rho} = \xi_\rho + \gamma_\rho \quad (26)$$

$$E_{B\rho} = \xi_\rho - \gamma_\rho \quad \text{and} \quad (27)$$

where $\xi_\rho = (E_3 + E_\rho)/2$, $\gamma_\rho = [(E_3 - E_\rho)^2/4 + \alpha_\rho^2]^{1/2}$ and $\rho = 1, 2$ for the symmetric and asymmetric channels, respectively. The time-dependent wave function $|\Psi(t)\rangle$ of the composite proton in the asymmetric three well potential can be expressed in terms of the corresponding eigenstates as

$$|\Psi(t)\rangle = |\Psi_{A1}\rangle \exp(-i E_{A1} t/\hbar) + |\Psi_{A2}\rangle \exp(-i E_{A2} t/\hbar) + |\Psi_{B1}\rangle \exp(-i E_{B1} t/\hbar) + |\Psi_{B2}\rangle \exp(-i E_{B2} t/\hbar), \quad (28)$$

which can be expressed in terms of physical base states $|3\rangle, |2\rangle, |1\rangle$ as [120]

$$|\Psi(t)\rangle = \exp(-i \xi_1 t/\hbar) \{ |3\rangle \exp(-i \gamma_1 t/\hbar) + |1'\rangle \exp[-i (\gamma_1 t/\hbar + \delta)] \} + \exp(-i \xi_1 t/\hbar) \{ |3\rangle \exp(+i \gamma_1 t/\hbar) + |1'\rangle \exp[+i (\gamma_1 t/\hbar + \delta)] \} + \exp(-i \xi_2 t/\hbar) \{ |3\rangle \exp(-i \gamma_2 t/\hbar) + |2'\rangle \exp[-i (\gamma_2 t/\hbar + \delta)] \} + \exp(-i \xi_2 t/\hbar) \{ |3\rangle \exp(+i \gamma_2 t/\hbar) + |2'\rangle \exp[+i (\gamma_2 t/\hbar + \delta)] \}. \quad (29)$$

This can be written more succinctly as

$$|\Psi(t)\rangle = (0.5)^{1/2} \exp(-i \xi_1 t/\hbar) \{ |3\rangle \cos(\gamma_1 t/\hbar) + |1'\rangle \sin(\gamma_1 t/\hbar) \} + (0.5)^{1/2} \exp(-i \xi_2 t/\hbar) \{ |3\rangle \cos(\gamma_2 t/\hbar) + |2'\rangle \sin(\gamma_2 t/\hbar) \} \quad (30)$$

where $|1\rangle = |1'\rangle e^{i\delta}$, $|2\rangle = |2'\rangle e^{i\delta}$ and δ of the arbitrary phase factor $e^{i\delta}$ is $-\pi/2$ and the relation $\cos(\theta - \pi/2) = \sin(\theta)$ is used. Data show that *ts* rates are approximately equal for transversions and transitions; so, quantum mechanical “rate constants” for EPR arrangements, *keto-amino* \rightarrow *enol-imine* via symmetric and asymmetric channels, are approximately equal (Figure 11). Since the lifetimes, τ , for 37°C keto-amino G-C protons are, $\tau \geq 3,000$ years, the wave function expression in Equation (30) is applicable in the interval, $0 < t < 3,000$ years[35-36,118-119].

At $t = 0$, the composite proton was in the metastable state $|3\rangle$ at energy E_3 . The probability, $P_1(t)$, that the proton is in the ground state $|1\rangle$, at a later time t , is given by

$$P_1(t) = |\langle 1' | \Psi(t) \rangle|^2 = 0.5 \sin^2(\gamma_1 t/\hbar) \quad (31)$$

which is the contribution by the asymmetric channel. The probability that the proton is in metastable state $|3\rangle$ at time t is given by

$$P_2(t) = |\langle 2' | \Psi(t) \rangle|^2 = 0.5 \sin^2(\gamma_2 t/\hbar) \quad (32)$$

which is the contribution by the asymmetric channel. The probability that the proton is in metastable state $|3\rangle$ at time t is given by

$$P_3(t) = |\langle 3 | \Psi(t) \rangle|^2 = 0.5 [\cos^2(\gamma_1 t/\hbar) + \cos^2(\gamma_2 t/\hbar)] \quad (33)$$

which is the sum of contributions for protons existing state $|3\rangle$ by the symmetric and asymmetric channels.

$$\sum_{i=1}^3 P_i(t) = (0.5) \left[\sin^2 \left(r_1 t / \hbar \right) + \cos^2 \left(r_1 t / \hbar \right) \right] + (0.5) \left[\sin^2 \left(r_2 t / \hbar \right) + \cos^2 \left(r_2 t / \hbar \right) \right] = 1, \quad (34)$$

is consistent with the requirement that the composite proton be confined to its set of base states, $|3\rangle$, $|2\rangle$, $|1\rangle$. The time derivative of $P_p(t)$, Equations (31 & 32), can be expressed as

$$dP_p/dt = (\gamma_p/\hbar) \sin(\gamma_p t/\hbar) \cos(\gamma_p t/\hbar) \quad (35)$$

where $P_p(t)$ represents either $P_1(t)$ or $P_2(t)$ and the 0.5 normalization factor is omitted. A Taylor series expansion of Equation (35) is given by

$$dP_p/dt \approx (\gamma_p/\hbar)^2 t - 4/3 (\gamma_p/\hbar)^4 t^3 + 4/15 (\gamma_p/\hbar)^6 t^5 + \dots \quad (36)$$

where the first three terms are given. The experimental lifetime of metastable keto-amino G-C protons is the order of ~3,000 years, which is large compared to human lifetimes of, say, ~100 years. For times $t \ll 3,000$ years (e.g., $t \leq 100$ years), one could employ a small t approximation to express the probability of metastable protons populating enol and imine states $|1\rangle$ or $|2\rangle$ as

$$P_p(t) = \frac{1}{2} (\gamma_p/\hbar)^2 t^2, \quad (37)$$

indicating *nonlinear* time dependence. Nonlinearity of Eq (37) is consistent with *exponential* increases observed in base substitutions, **ts**, and deletions, **td**, as a function of age in *nonmitotic* human mtDNA [121]. Equation (37)

The sum of Equations (31 to 33), given by

provides the approximate quantum entanglement term for “biological noise” in Eq (17), and expresses observable quantum contributions in the Darwinian EPR-entanglement polynomial, $\Sigma_i \beta_i t^4$ in Eq (22). For times $t \leq 100$ y, this approximation accounts for the time-dependent generation of quantum informational content embodied within entangled proton qubits populating decoherence-free subspaces of **G'-C'**, ***G-*C** and ***A-*T** sites in duplex genomic systems [16-19,23-25,35-36,44]. Subsequent enzyme – proton entanglement processing of entangled proton qubits introduces entanglement originated “*stochastic*” mutations, **ts** and **td**, which are expressed in terms of $\Sigma_i \beta_i t^4$ in the Darwinian polynomial, Eq (22). Curiously, incidence of age-related (10 to 80 y) human cancer [77] exhibits an empirical $\sim t^4$ time-dependence (Figure 7), implying phenotypic expression of age-related cancer is a consequence of the quantum entanglement algorithm, yielding decohered products, **ts** and **td**. Based on observation and the model, Eq (22), the EPR-generated quantum entanglement algorithm introduces single nucleotide polymorphisms, **SNPs**, which are expressed as cancer causing “driver” mutations *after* CNGS, s ($1 \geq s \geq 0.97$), have been populated by entangled proton qubits to an “unsafe” threshold, i.e., to $s \approx 0.97 + \varepsilon$ [16-19,42,77-80,87-88]. In this case, classical “ball-and-rod” Newtonian mechanisms do *not* contribute to “driver” mutation spectra for age-related incidence of cancer exhibited in (Figure 7) [48,55,67,89].