



Radiatively Induced Finite and (Un) Determined Chern-Simons-Like Terms

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Abstract

The problem of Chern-Simons-like term induction via quantum corrections in four-dimensions is investigated in two different cases. In the first case, we consider two distinct approaches to deal with the exact fermion propagator of the extended QED theory up to the first order in the b -coefficient. We find different results for distinct approaches in the same regularization scheme. In the second case, we show that when we use a modified derivative expansion method and another regularization scheme, we obtain a result that exactly coincides with one of the results obtained in the former case. This seems to imply an ambiguity absence as one treats the fermion propagator and the self-energy tensor properly.

Keywords: Chern-Simons Theories; Space-Time Symmetries; Quantum Corrections

Introduction

The induction of the Chern-Simons-like Lorentz and CPT violating term, given by $\mathcal{L}_{CS} = \frac{1}{2} k_\mu \epsilon^{\mu\nu\lambda\theta} F_{\nu\lambda} A_\theta$, being k_μ a constant vector characterizing the preferred direction of the space-time, is one of the most important results in the study of Lorentz symmetry violation [1,2]. This term which is known to have some important implications, such as birefringence of light in the vacuum [3], naturally emerges as a quantum correction in the theory suggested in Jackiw R [3] as a possible extension of QED:

$$\mathcal{L}_{QED} = \bar{\psi}(i\partial - m)\psi - \bar{\psi}b\gamma_5\psi - e\bar{\psi}A\psi, \quad (1)$$

Where b_μ is a parameter introducing CPT symmetry breaking? Carrying out the integration over fermions, the relation between the coefficients k_μ and b_μ could be

obtained in terms of some loop integrals with some of them being divergent. Therefore, one has to implement some regularization scheme to calculate these integrals. Thus, the constant relating the coefficient k_μ and b_μ turns out to be dependent on the regularization scheme used [4,5]. Such dependence on the regularization scheme has been intensively discussed in Jackiw R [6] Chung M [7] Perez-Victoria M [8]. However, there is an alternative study with absence of ambiguities put forward recently in Bonneau G [9].

Based on the theory (1), the purpose of our study is to investigate different possibilities of finding ambiguities inherent to generation of Chern-Simons-like term via quantum corrections in four dimensions. We do this by using derivative expansion method of the fermion determinant [10] and the imaginary time formalism.

The work structure is organized as follows: In section 2 we will investigate the induction of the Chern-Simons-like term via quantum corrections by using distinct approaches to deal with the exact fermion propagator. We find distinct relations between the coefficients k_μ and b_μ for different approaches, but with a same regularization scheme. Therefore, we conclude that different approaches to deal with the exact fermion propagator of the theory, we conclude that different approaches to deal with the exact fermion propagator of the theory leads to a new ambiguity for the problem of radiatively induced Chern-Simons-like term. In section 3 we develop another method to investigate the present issue. By modifying the derivative expansion method, we obtain a different self-energy tensor. We then use a specific regularization scheme to find a finite result identical to that obtained in section 2 by using another regularization scheme. This effect seems to imply ambiguity absence in our calculations. Finally, in section 4 we present our conclusions.

Inducing Chern-Simons-like Term: Two different Approaches

In this section, we focus on the induction of the Chern-Simons-like term coefficient by expanding the self-energy and using two distinct approaches to deal with the exact fermion propagator up to the leading order in b . We find a new ambiguity because two different results appear.

The one-loop effective action $S_{eff}[b, A]$ of the gauge field A_μ related to theory (1), can be expressed in the form of the following functional trace:

$$S_{eff}[b, A] = -iTr \ln(\not{p} - m - \not{b}\gamma_5 - e\mathcal{K}) . (2)$$

This functional trace can be represented as $S_{eff}[b, A] = S_{eff}[b] + S'_{eff}[b, A]$, where the first term $S_{eff}[b] = -iTr \ln(\not{p} - m - \not{b}\gamma_5)$ does not depend on the gauge field. The only nontrivial dynamics is concentrated in the second term $S'_{eff}[b, A]$, which is given by the following power series:

$$S'_{eff}[b, A] = iTr \sum_{n=1}^{\infty} \frac{1}{n} \left[\frac{1}{\not{p} - m - \not{b}\gamma_5} e\mathcal{K} \right]^n . (3)$$

To obtain the Chern-Simons-like term we should expand this expression up to the second order in the gauge field:

$$S'_{eff}[b, A] = S_{eff}^{(2)}[b, A] + \dots . (4)$$

The dots in (4) stand for higher order terms in the gauge field. Here

$$S_{eff}^{(2)}[b, A] = \frac{ie^2}{2} Tr \left[\frac{1}{\not{p} - m - \not{b}\gamma_5} \not{A} \frac{1}{\not{p} - m - \not{b}\gamma_5} \not{A} \right] . (5)$$

Using the derivative expansion method [10] one can find that the one-loop contribution to $S_{eff}^{(2)}[b, A]$ reads

$$S_{eff}^{(2)}[b, A(x)] = \frac{1}{2} \int d^4x \Pi^{\alpha\mu\nu} F_{\alpha\mu} A_\nu , (6)$$

where the one-loop self-energy $\Pi^{\alpha\mu\nu}$ is given by

$$\Pi^{\alpha\mu\nu} = -\frac{ie^2}{2} \int \frac{d^4p}{(2\pi)^4} tr[S_b(p)\gamma^\mu S_b(p)\gamma^\alpha S_b(p)\gamma^\nu] , (7)$$

Where

$$S_b(p) = \frac{i}{\not{p} - m - \not{b}\gamma_5} (8)$$

is a b^μ dependent exact fermion propagator of the theory.

Approach I: Fermion Propagator Rationalized

Firstly, we use the approximation developed in Ebert D, et al. [11], where the exact propagator (8) is rationalized in the form

$$S_b(p) = i \left[\frac{\not{p} + m - \not{b}\gamma_5}{(p^2 - m^2)} - \frac{2\gamma_5(m\mathcal{K} - (b \cdot p))(\not{p} + m)}{(p^2 - m^2)^2} \right] + \dots (9)$$

Substituting (9) into (7), we can calculate the trace of gamma matrices, resulting in the following expression for the self-energy tensor [4]:

$$\Pi_r^{\mu\alpha\nu} = -2ie^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 + m^2)^3} \{ 3\varepsilon^{\alpha\mu\nu\theta} [b_\theta(p^2 - m^2) - 2p_\theta(b \cdot p)] - 2_0[\varepsilon^{\beta\mu\nu\theta} p_\beta p^\alpha + \varepsilon^{\alpha\beta\nu\theta} p_\beta p^\mu + \varepsilon^{\alpha\mu\beta\theta} p_\beta p^\nu] \} , (10)$$

Where $\Pi_r^{\mu\alpha\nu}$ means self-energy tensor is rationalized up to the first order in b -coefficient. In Equation (10), we turn the Minkowski space to a Euclidean space by performing the Wick rotation $x_0 \rightarrow -ix_0, p_0 \rightarrow ip_0, b_0 \rightarrow ib_0, d^4x \rightarrow -id^4x$ and $d^4p \rightarrow id^4p$. Note that by power counting, the momentum integral in (10) involves a finite term and terms with logarithmic divergences. In order to regularize such divergence, we use a scheme which we implement translation only on space coordinates of the momentum p_ρ [12-14]. Hence, we have

$$p_\rho \rightarrow \vec{p}_\rho + p_0 \delta_{0\rho} . (11)$$

We use the covariance under spatial rotations which allows us to carry out the following replacement

$$\vec{p}_\rho \vec{p}^\sigma \rightarrow \frac{\vec{p}^2}{D} (\delta_\rho^\sigma - \delta_{\rho 0} \delta_0^\sigma) . (12)$$

Thus,

$$2p_\rho(b.p) \rightarrow 2\left(b_\rho \frac{\bar{p}^2}{D} - b_0 \delta_{\rho 0} \left(\frac{\bar{p}^2}{D} - p_0^2\right)\right), \quad (13)$$

$$2p_\beta p^\alpha \rightarrow 2\left(\delta_\beta^\alpha \frac{\bar{p}^2}{D} - \delta_{\beta 0} \delta_0^\alpha \left(\frac{\bar{p}^2}{D} - p_0^2\right)\right).$$

We have that only the terms above can contribute to the Chern-Simons structure. Therefore, we can split the expression (5) into a sum of two parts, “covariant” and “noncovariant”, i.e.,

$$S_{eff}^{cov} = \frac{(-i)}{2} \int d^4 x I_1 \varepsilon^{\alpha\mu\nu\beta} b_\beta F_{\alpha\mu} A_\nu, \quad (14)$$

With

$$I_1 = \frac{-3ie^2}{2\pi} \int \frac{d^D \bar{p}}{(2\pi)^D} \int_{-\infty}^{+\infty} dp_0 \left[\frac{\left(1 - \frac{4}{D}\right) \bar{p}^2 + p_0^2 - m^2}{(\bar{p}^2 + p_0^2 + m^2)^3} \right], \quad (15)$$

And

$$S_{eff}^{ncv} = \frac{i}{2} \int d^4 x I_2 [3\varepsilon^{\alpha\mu\nu\theta} b_0 F_{\alpha\mu} A_\nu + b_\theta (\varepsilon^{0\mu\nu\theta} F_{0\mu} A_\nu + \varepsilon^{\alpha 0\nu\theta} F_{\alpha 0} A_\nu + \varepsilon^{\alpha\mu 0\theta} F_{\alpha\mu} A_0)], \quad (16)$$

With

$$I_2 = \frac{ie^2}{\pi} \int \frac{d^D \bar{p}}{(2\pi)^D} \int_{-\infty}^{+\infty} dp_0 \left[\frac{\frac{\bar{p}^2}{D} - p_0^2}{(\bar{p}^2 + p_0^2 + m^2)^3} \right]. \quad (17)$$

The integrals I_1 and I_2 over p_0 are finite and can be calculated by residues theorem. Hence, we have

$$I_1 = \frac{3ie^2}{8D} \int \frac{d^D \bar{p}}{(2\pi)^D} \left[\frac{2(3-D)\bar{p}^2 + Dm^2}{(\bar{p}^2 + m^2)^{3/2}} \right], \quad (18)$$

And

$$I_2 = \frac{ie^2}{8D} \int \frac{d^D \bar{p}}{(2\pi)^D} \left[\frac{(3-D)\bar{p}^2 - Dm^2}{(\bar{p}^2 + m^2)^{5/2}} \right]. \quad (19)$$

Now, we can integrate over the spatial momentum in D-dimensions [15], and we find,

$$I_1 = \frac{3ie^2}{8(4\pi)^{D/2}} \frac{\Gamma\left(1 + \frac{\varepsilon}{2}\right)}{\Gamma\left(\frac{5}{2}\right) (m^2)^{\varepsilon/2}}, \quad (20)$$

where $\varepsilon = 3 - D$. The integral I_2 presents a value identically equal to zero. Therefore, the effective action Equation (14) in the limit $D = 3$ can be written in the form

$$S_{eff}^{cov} = \frac{1}{2} \int d^4 x \varepsilon^{\alpha\mu\nu\beta} \kappa_\beta F_{\alpha\mu} A_\nu, \quad (21)$$

Where

$$\kappa_\beta = \frac{e^2}{16\pi^2} b_\beta, \quad (22)$$

that coincides with the resulted of work Chan [16] in the Schwinger constant field approximation.

Approach II: Fermion Propagator Expansion

Now we use the approximation developed in Jackiw R, et al. [13] to expand the exact propagator (8) up to the first order in b -coefficient

$$S_b(p) = \frac{i}{\not{p} - m} + \frac{i}{\not{p} - m} (-i\cancel{B}\gamma_5) \frac{i}{\not{p} - m} + \dots \quad (23)$$

In this case, we have our self-energy tensor in the form [5,17]:

$$\Pi_e^{\mu\alpha\nu} = -2ie^2 \varepsilon^{\mu\alpha\nu\rho} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 + m^2)^3} \quad (24)$$

$$\times [b_\rho (p^2 - 3m^2) - 4p_\rho (b.p)],$$

Where $\Pi_e^{\mu\alpha\nu}$ means the expanded self-energy tensor. In Equation (24), we also change the Minkowski space to Euclidean space. Note that by power counting, the integral in the momentum space Equation (24) also present a finite term and another that diverges logarithmically. Thus, we use the same regularization scheme adopted in the rationalized fermion propagator approach. In this way, we obtain the following effective action in the limit of $D = 3$;

$$\tilde{S}_{eff}^{cov} = \frac{1}{2} \int d^4 x \varepsilon^{\alpha\mu\nu\beta} \tilde{\kappa}_\beta F_{\alpha\mu} A_\nu, \quad (25)$$

Where

$$\tilde{\kappa}_\beta = \frac{e^2}{4\pi^2} b_\beta. \quad (26)$$

Here, we also have the absence of “noncovariant” part. The result (26) is equivalent to the obtained in Andrianov AA, et al. [18] where one uses a physical cutoff for fermions. According to the results (22) and (26), we found that the use of different approaches to deal with the exact fermion propagator leads to distinct relations between the coefficient

κ_μ and b_μ for the Chern-Simons-like term. Our results establish the following relation:

$$\tilde{\kappa}_\beta = k_\beta + \frac{3e^2}{16\pi^2} b_\beta, \quad (27)$$

that means

$$\Delta k_\beta = \frac{3e^2}{16\pi^2} b_\beta. \quad (28)$$

The result (28) corresponds to variation between the results (22) and (26), that is identical to the result of References. Mariz T, et al. [17] originated from the self-energy (24), in another regularization scheme. The same result has been found in the literature Fraser [13], Vainshtein, et al. [14].

Other Aspects on Induced Chern-Simons-Like Term

In this section, we present an alternative method to compute induced Chern-Simons-like term that is independent of the approach used to deal with the exact fermion propagator. Let us rewrite the Equation (3) in the form,

$$S'_{eff}[b, A] = iTr \sum_{n=1}^{\infty} \frac{1}{n} \left[\frac{1}{\not{p} - m - eA} \not{K} \gamma_5 \right]^n. \quad (29)$$

To obtain the Chern-Simons-like term we should expand this expression up to the leading order in b . Thus, for $n=1$, we have

$$S_{eff}^{(1)}[b, A] = iTr \left[\frac{1}{\not{p} - m - eA} \not{K} \gamma_5 \right]. \quad (30)$$

Using the relation

$$\frac{1}{A-B} = \frac{1}{A} + \frac{1}{A} B \frac{1}{A} + \frac{1}{A} B \frac{1}{A} B \frac{1}{A} + \dots, \quad (31)$$

For $A = \not{p} - m$ and $B = eA$, we find

$$S_{eff}^{(1)}[b, A] = ie^2 Tr \left[\frac{1}{\not{p} - m} \not{K} \gamma_5 \frac{1}{\not{p} - m} A \frac{1}{\not{p} - m} A \right], \quad (32)$$

where one was considered the cyclic property of the trace in the product of the γ -matrices. By using derivative expansion method, we find the following effective action:

$$S_{eff}^{(1)}[b, A(x)] = \frac{1}{2} \int d^4 x \Pi^{\alpha\mu\nu} F_{\alpha\mu} A_\nu, \quad (33)$$

where the one-loop self-energy $\Pi^{\alpha\mu\nu}$ is given by

$$\Pi^{\mu\alpha\nu} = -2ie^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - m^2)^3} \left\{ \varepsilon^{\alpha\mu\nu\theta} b_\theta (p^2 - m^2) - 2b_\theta \left[\varepsilon^{\alpha\nu\theta\beta} p_\beta p^\mu - \varepsilon^{\alpha\nu\theta\beta} p_\beta p^\nu \right] \right\}. \quad (34)$$

Note that in the self-energy tensor (34) there exists a convergent contribution and the remaining term diverges logarithmically. However, differently of the previous situations, in this case the calculation of the divergent integrals is very delicate. Calculating this self-energy tensor by using the same regularization scheme of the section 2, we find a null result, and in turn, the absence of the Chern-Simons term. On the other hand, there also exists the possibility of using in Equation (34) the relation

$$\int \frac{d^4 p}{(2\pi)^4} p_\mu p_\nu f(p^2) = \frac{g_{\mu\nu}}{4} \int \frac{d^4 p}{(2\pi)^4} p^2 f(p^2), \quad (35)$$

that naturally removes the logarithmic divergence. As a result, we have only the finite contribution

$$\Pi^{\mu\alpha\nu} = \frac{e^2}{16\pi^2} \varepsilon^{\mu\alpha\nu\theta} b_\theta. \quad (36)$$

In this case, we find

$$S_{eff}^{(1)}[b, A(x)] = \frac{1}{2} \int d^4 x k_\beta \varepsilon^{\mu\alpha\nu\beta} \partial_\alpha A_\mu A_\nu, \quad (37)$$

Where

$$k_\beta = \frac{e^2}{16\pi^2} b_\beta. \quad (38)$$

Note that the result (38) is the same as the result found in (22) where the exact fermion propagator was rationalized up to the first order in the b -coefficient. Thus, we have here another surprising effect: The result $k_\beta = \frac{e^2}{16\pi^2} b_\beta$ given in

(22) as result of dimensional regularization and also given in (38) as result of the Lorentz preserving regularization (35), appears to be independent of the regularization scheme used by properly carrying out the self-energy tensor. The result (38) was also obtained in the work by Perez-Victoria M [14] using massless exact fermion propagator. We observe that the factor $(e^2/16\pi^2)$, is exactly the same as found in the well-known Adler-Bell-Jackiw anomaly [19-21].

Conclusion

We have investigated the induction of Chern-Simons-like term via quantum corrections in two different situations. Firstly, we use a same regularization to different approaches to deal with the exact fermion propagators up to the leading order in the b -coefficient. In this case, our results are finite and agree with other results in the literature, but they do not

agree with each other, because they are different depending on the approach used. Moreover, it generates values whose difference is exactly: $\Delta k_\beta = (3e^2/16\pi^2)b_\beta$, which agrees with the result of References. Mariz T, et al. [17] found in another context. We conclude that this is due to the different approximation of the exact fermion propagator of theory. The problem was also investigated in a context independent of the approaches used to deal with the exact fermion propagator. In this case, we modify the derivative expansion method and obtain a new self-energy tensor for the effective action. The momentum integrals were calculated by using another regularization scheme. As a result, we obtained a relation between the coefficients k_β and b_β identical to the one obtained in the case where it was used the fermion propagator expansion. We also observed that the parameter of proportionality between these two coefficients is exactly the same as the one found in the well-known Adler-Bell-Jackiw anomaly. In our calculations, we also observed that the “noncovariant” contributions for the Chern-Simons-like term are absent, as was anticipated in Gomes M, et al. [4,22-32] in the finite temperature context. Therefore, we insist that a complete comprehension of this question will require further investigations.

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References

- Carroll SM, Field GB, Jackiw R (1990) Limits on a Lorentz- and parity-violating modification of electrodynamics. *Phys Rev D* 41(4-15): 1231.
- Colladay D, Kostelecký VA (1997) CPT violation and the standard model. *Phys Rev D* 55(11-1): 6760.
- Jackiw R (2002) Physical instances of noncommuting coordinate. *Nucl Phys B Proc Suppl* 108: 30-36.
- Brito FA, Nascimento JR, Passos E, Petrov AY (2007) Lorentz-CPT violation, radiative corrections and finite temperature. arXiv: 0705.1338v2.
- Gomes M, Nascimento JR, Petrov AY, Silva DAJ (2010) Aetherlike Lorentz-breaking actions. *Phys Rev D* 81: 045018.
- Jackiw R (1999) When radiative corrections are finite but undetermined. *High Energy Physics - Theory* 14(16): 49.
- Chung JM (1999) Lorentz- and CPT-violating Chern-Simons term in the functional integral formalism. *Phys Rev D* 60: 127901.
- Perez-Victoria M (2001) Physical (ir)relevance of ambiguities to Lorentz and CPT violation in QED. *JHEP* 04: 032.
- Bonneau G (2001) Regularisation: many recipes, but a unique principle: Ward identities and normalisation conditions. The case of CPT violation in QED. *Nucl Phys B* 593(1-2):398-412.
- Brito FA, Nascimento JR, Passos E, Petrov AY (2007) Lorentz-CPT violation, radiative corrections and finite temperature. *High Energy Physics - Theory*.
- Aitchison IJR, Fraser CM (1984) Fermion loop contribution to skyrmion stability. *Phys Lett B* 146(1-2): 63-66.
- Aitchison IJR, Fraser CM (1985) Derivative expansions of fermion determinants: Anomaly-induced vertices, Goldstone-Wilczek currents, and Skyrme terms. *Phys Rev D* 31: 2605.
- Fraser CM (1985) Calculation of higher derivative terms in the one-loop effective Lagrangian. *Zeitschrift für Physik C Particles and Fields* 28: 101-106.
- Vainshtein AI, Zakharov VI, Novikov VA, Shifman MA (1984) Calculations in external fields in quantum chromodynamics. Technical review. *Fortschritte der Physik* 32(11): 585-622.
- Zuk JA (1985) Functional approach to derivative expansion of the effective Lagrangian. *Phys Rev D* 32: 2653.
- Chan LH (1985) Effective-Action Expansion in Perturbation Theory. *Phys Rev Lett* 54(1-2): 1222.
- Gaillard MK (1985) The Effective One Loop Lagrangian With Derivative Couplings. *Nucl Phys B* 268: 669-692.
- Das A, Kitev A (1987) Derivative expansion and the chiral anomaly at finite temperature. *Phys Rev D Part Fields* 36(2): 623-626.
- Babu KS, Das A, Panigrahi P (1987) Derivative expansion and the induced Chern-Simons term at finite temperature in 2+1 dimensions. *Phys Rev D Part Fields* 36(12): 3725-3730.
- Ebert D, Zhukovsky VCH, Razumovsky AS (2004) Chern-

- Simons-like term generation in an extended model of QED under external conditions. *Phys Rev D* 70: 025003.
21. Cervi L, Griguolo L, Seminara D (2001) Structure of radiatively induced Lorentz and CPT violation in QED at finite temperature. *Phys Rev D* 64: 105003.
 22. Jackiw R, Kostelecký VA (1999) Radiatively Induced Lorentz and CPT Violation in Electrodynamics. *Phys Rev Lett* 82: 3572.
 23. Perez-Victoria M (1999) Exact Calculation of the Radiatively Induced Lorentz and CPT Violation in QED. *Phys Rev Lett* 83: 2518.
 24. Hooft G, Veltman MJG (1972) Regularization and Renormalization of Gauge Fields. *Nucl Phys B* 44: 189-213.
 25. Chaichian M, Chen WF, González Filipe R (2001) Radiatively induced Lorentz and CPT violation in Schwinger constant field approximation. *Phys Lett B* 503(1-2): 215-222.
 26. Brito FA, Mariz T, Nascimento JR, Passos E, Ribeiro RF (2005) A Remark on Lorentz Violation at Finite Temperature. *JHEP* 10: 019.
 27. Andrianov AA, Giacconi P, Soldati R (2002) Lorentz and CPT violations from Chern-Simons modifications of QED. *JHEP* 02: 030.
 28. Adler SL (1969) Axial-Vector Vertex in Spinor Electrodynamics. *Phys Rev* 177: 2426.
 29. Adler SL, Bardeen WA (1969) Absence of Higher-Order Corrections in the Anomalous Axial-Vector Divergence Equation. *Phys Rev* 182: 1517.
 30. Bell JS, Jackiw R (1969) A PCAC puzzle: $\pi^0 \rightarrow \gamma\gamma$ in the σ -model. *Nuovo Cimento A* (1965-1970) 60: 47-61.
 31. Peskin ME, Schroeder DV (1995) An Introduction to Quantum Field Theory. Persus Books, pp: 1-864.
 32. Banerjee H, Asit KD (1996) Fermions on lattice and chiral invariance. *High Energy Physics – Theory*.

