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Research of Theory on High Temperature Superconductivity, Super Fluidity and BEC

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Abstract

First, some progress of high-temperature superconductivity is discussed. Then we research its theory, including the extensive quantum theory, soliton solution, bag model and so on. Third, superfluidity and Bose-Einstein condensation (BEC), etc., are searched, such as the double solution of soliton-chaos in nonlinear equations. Finally, we predict existences of the high-temperature superfluidity and BEC, etc., and discuss some problems.

Keywords: High- T_c superconductivity; Superfluidity; BEC; Quantum theory; Nonlinearity; Bag

Introduction

Many macroscopic quantum phenomena exist widely [1]. They include superconductivity, superfluidity and Bose-Einstein condensation (BEC), etc., in which the order parameter is a complex field, and can be described by similar theoretical ideas [2]. All condensed state physics is result of breaking symmetry. It is always an important direction of experiment and theory of modern physics.

In the traditional superconducting materials, the dominant force is relatively single, which greatly simplifies the theoretical solution. However, in the high temperature superconductivity system, the lattice vibration, Coulomb force, magnetic coupling, kinetic energy and so on influence each other, so the strong correlation effects between electrons increase the complexity of the system. Some basic theories obtained by Hubbard model, T-J model, etc., can only agree with part of the experimental results. So far, the specific mechanism of high-temperature superconductivity is still unclear. Following the mechanism of BCS pairing is still controversial [3,4]. It is generally believed that electron interactions can be divided into lattice vibrations based on BCS theory, and antiferromagnetic spin excitation. Oxide high-temperature superconducting materials are typically strongly associated systems, and are highly anisotropic materials. Its physical properties are dominated by the strong correlation effect of electrons in the copper-oxygen plane shared in these materials, which are related to strong quantum fluctuations, and are completely different from the Fermi liquid [5-8]. Universal is that the superconducting ground states are all formed by the Bose condensation of the electron Cooper pair.

Moreover, the potassium-doped C60 has superconductivity at low temperature 18K [9], which indicates that superconductivity can have multiple structures. In this paper, we research that high- T_c superconductivity, superfluidity, BEC may be described by the extensive quantum theory and so on, and propose some predictions on macroscopic quantum phenomena.

Some Progress of High Temperature Superconductivity

At present, the common high-temperature superconductor materials are: copper oxide superconductors, cobalt oxide superconductors and heavy fermion superconductors, etc. After 2008, superconducting materials with iron and arsenic layer (FeAs) or iron and selenium layer (FeSe) appeared as the main structural units. It becomes the second type of high temperature superconductors after copper oxide. Moreover, iron-based superconductors are similar to copper oxide superconductors, with quasi-two-dimensional layered structure, and the crystal reflects strong two-dimensional characteristics, superconducting properties have strong anisotropy. This suggests that the both may be a common origin of high temperature superconductivity.

In 2015 Drozdov and Eremets, et al. found that when the HS is compressed to nearly 2 million times the atmospheric pressure, and realized superconductivity at 203K (70°C) [10], and lanthanide hydroxide (LaH10) becomes a superconductor at 250K (23°C) and at 170 GPa high pressure [11].

In 1964 Little proposed the concept of one-dimensional polymer organic superconductors. He believes that superconductors may be present in one-dimensional organic polymers, and that the superconducting transition temperature is much higher than at room temperature. From this in 2020 Ranga P. Dias, et al. combine hydrogen-rich materials with organic superconductivity by substituting carbon for metal elements. A uniform and transparent C-S-H crystal structure was produced, and the C-S-H superconducting system at 288K (15°C) under 267GPa [12]. This is the first time that an organic superconducting system contains three elements rather than two.

For the superconductivity phenomenon of hydrogen-rich materials at high pressure, theoretical physicists have launched a heated discussion and prediction: in some cases, elements with low atomic mass may lead to high critical temperatures. Errea, et al. discussed quantum crystal structure in the 250-kelvin superconducting lanthanum hydride [13]. Wang, et al. researched future study of dense superconducting hydrides at high pressure [14]. Hydrogen is the lightest element, and the hydrogen bond is one of the strongest chemical bonds. The hydrogen ion lattice in hydrogen sulfide can vibrate quickly at higher temperatures, and transport Cooper electron pairs unimpeded. Under the action of high pressure, the hydrogen ion lattice can maintain a strong structure. Such a room-temperature superconductivity may lie in the hydrogen-rich add organic superconductivity. Moreover, fine-tuning the components of the C-S-H ternary system by exchanging molecules at lower pressures is expected to achieve stable or sub-stable high-temperature superconductor at atmospheric pressure. Further, the addition of a third element could greatly broaden the scope of future searches for new superconductors.

Pfleiderer, et al. confirmed the superconductivity and ferromagnetic unified in the d-band metal ZrZn2 by experiments [15]. Aoki, et al. confirmed coexistence of superconductivity and ferromagnetism in UrhGe [16]. Chen, et al. searched superconductivity at 41K and its competition with spin-density-wave instability in layered $CeO_{1-X}F_XFeAs$ [17]. Rotter, et al. investigated superconductivity at 38K in the iron arsenide $Ba_{1-x}K_xFe_2As_2$ [18]. Ni, et al. searched anisotropic thermodynamic and transport properties of single crystalline $Ba_{1-x}K_xFe_2As_2$ [19]. Yuan, et al. investigated nearly isotropic superconductivity in (Ba,K) Fe_2As_2 [20]. Zabolotnyy, et al. researched (π, π) electronic order in iron arsenide superconductors [21].

Recently, Chen, et al. discussed the $LaAlO_3/KTaO_3$ (110) heterointerface in two dimensional superconductivity [22]. Li, et al. discussed topological transition of superconductivity in Dirac semimetal nanowire Josephson junctions [23]. Peri, et al. studied fragile topology and flat-band superconductivity in the strong-coupling regime [24]. Lee, et al. studied odd-parity spin-triplet superconductivity in centrosymmetric antiferromagnetic metals [25]. Zhang and Sarma searched intrinsic time-reversal-invariant topological superconductivity in thin films of iron-based superconductors [26]. Collomb, et al. observed the suppression of superconductivity in $RbEuFe_4As_4$ by correlated magnetic fluctuations [27]. Xu and Grover searched competing nodal d-wave superconductivity and antiferromagnetism [28]. Zhang, et al. searched anomalous Floquet chiral topological superconductivity in a topological insulator sandwich structure [29]. Jiang, et al. searched high temperature superconductivity in a lightly doped quantum spin liquid [30]. Gong, et al. searched robust d-wave superconductivity in the square-lattice t-J model [31]. Can, et al. probed time reversal symmetry breaking topological superconductivity in twisted double layer copper oxides with polar Kerr effect [32]. Jian, et al. studied charge-4e superconductivity from nematic superconductors in two and three dimensions [33]. Shavit, et al., discussed theory of correlated insulators and superconductivity in twisted bilayer grapheme [34].

In 2022 Lou, et al. researched charge-density-wave-induced peak-dip-hump structure and the multiband superconductivity in a Kagome superconductor CsV3Sb5 [35]. Steiner, et al. searched quantum magnetism and topological superconductivity in Yu-Shiba-Rusinov chains [36]. Liu, et al. studied quantum criticality of antiferromagnetism and superconductivity with relativity [37].

For the high-T_c superconductivity, some new quasiparticles, holon and spinon, etc., are proposed. Schrieffer, et al., discussed dynamical spin fluctuations and the bag mechanism [38], and the spin-bag approach [39]. Tao, et al., researched the formation of high temperature superconducting balls [40]. Hill, et al. discussed a breakdown of Fermi-liquid theory in a copper-oxide superconductor [41].

Metz, et al. proposed an entanglement generation scheme through macroscopic quantum jumps [42]. Ovchinnikov, et al. studied the phenomenon of macroscopic quantum tunneling in small Josephson junctions in a magnetic field [43]. Yu, et al. observed quantum jumps between macroscopic quantum states of a superconducting phase qubit coupled to a microscopic two-level system in the Josephson tunnel junction [44]. Buhmann and Scheel discussed that the duality between electric and magnetic fields is a valid symmetry of macroscopic quantum electrodynamics [45]. Martini, et al. shown that all macroscopic quantum superpositions based on phase-covariant quantum cloning are characterized by an anomalous high resilence to the decoherence processes [46]. Solenov and Mozyrsky studied metastable states and macroscopic quantum tunneling in a cold-atom Josephson ring [47]. Frowis and Dur studied the stable macroscopic quantum superpositions under decoherence [48]. Fedorov, et al. studied tuned transition from quantum to classical for macroscopic quantum states [49]. Based on phase-space structures of quantum states, Lee and Jeong proposed a novel measure to quantify macroscopic quantum superpositions [50].

New Research on Theory of High-T_c Superconductivity

So far the theoretical interpretation of high temperature superconductivity remains an unresolved core problem in modern physics [2]. Anderson believes that the normal state of high temperature oxide superconductor is the non-Fermi liquid, where the spin of electrons is separated from the charge element excitation, and the interlayer Josephson tunneling deconfinement as driving force derives high temperature of superconductivity. This suggests a corresponding Josephson effect for the high-temperature superconductivity. Laughlin agrees with the core idea of Anderson theory, namely the failure of the high temperature oxide superconductor for Fermi liquid principle [51]; he proposed the high temperature superconductivity theory: the ideal gas obeying the fractional statistics particles is a new superconductor [52]. Existing Fermi liquid picture composed of quasiparticles, which obeys Fermi statistics. It should be further developed. This may be the non-Fermi liquid proposed by Anderson, or the Fermi liquids have different intrinsic structures.

The Landau energy level is quantized:

$$E=(n+\frac{1}{2})\hbar\omega_c+\frac{\hbar^2 k_z^2}{2m^*} \quad (1)$$

It may relate to De Haas-von Alphen effect, Sunbnikov-de Haas effect, quantum Hall effect and so on in the magnetic field. Moreover, there are various quantized phonon, exciton, polaron, plasmon, quantum dot and quantum well, the devil staircase in locked frequency, etc.

In 1961 Goldstone, et al., proved that field theory has superconductivity solution, therefore theoretically modify field theory or its process, or conclusions may derive high temperature superconductivity. Far from the equilibrium the long-range correlation and its enhancement can contact and describe the high temperature superconductivity. This is a phase transition in which electron interactions are known or bind strong interactions, or new interactions. Superconductivity should be the combination of microscopic quantum theory and many-body electromagnetic interaction, stochastic theory, etc.

For theory of high temperature superconductivity, in 1987 Zhang Li-yuan proposed two subsystem assembly, which has mixing interaction. This may combine the Cooper pair and the Ogg pair in the high-T_c superconductor [53].

High temperature superconductivity has some new characters. Tao, et al. found formation of superconducting balls [54]. This is the very short coherence length with high temperature superconductivity, where electrons at very close distance may pair each other. This may inspire for mechanics of high-temperature superconductivity. Quasi-two-dimensionality is one of the important features of the high-temperature copper oxide superconductor [22,39]. Two dimensions correspond to anyon, so the relations between anyon and high temperature superconductivity should be researched.

We research three theories of high-T_c superconductivity: the extensive quantum theory [55-57], the soliton model in nonlinear equations and the bag model.

Great physicist Feynman pointed out: "There are certain situations in which the peculiarities of quantum mechanics can come out in a special way on large scale." In a special situation "quantum mechanics will produce its own characteristic effects on a large or 'macroscopic' scale" [58]. We proposed the extensive quantum theory [55-57], in which the formulations are the same with the quantum mechanics and only quantum constant $\hbar \to \mathrm{H}$ and corresponding basic quantum elements are different [55-57]. Further, it is developed to the extensive quantum biology [59-61].

The high-temperature superconductivity theories apply the known fermions and bosons, and also propose the new quasi-particle, holon and spinon, etc. Experimental results confirm that high temperature superconductor does not apply the authoritative Landau theory on Fermi liquid of electrons in metals [62]. It is connected to the fermion condensate, and negates the classical BCS theory. However, the interaction of the BCS theory is weak, and condition is $\ln\left(\hbar\omega_D / T_c\right) >> 1$. When the critical temperature T_c of superconductivity becomes larger, $\hbar\omega_D / T_c$ is smaller, so that the extensive quantum theory h→H is the simplest way [56]. In 1964 Little first proposed a strong coupling mechanism of superconductivity [63].

According to BCS theory, the critical temperature of superconductivity is:

$$T_c = \frac{1.14\hbar}{k}\omega_D \exp\left[-1 / N\left(0\right)V\right] \quad (2)$$

Here ω_D is Debey frequency. The highest temperature T_c=30K. It determines that the interaction strength N(0) V is weak. McMillan proposed the transition temperature formula for the strongly coupled superconductivity [64]:

$$T_c = \omega_0 \exp\left[\frac{-\left(1+\lambda\right)}{\lambda - \mu^* - \left(<\omega>/\omega_0\right)\lambda\mu^*}\right]. \quad (3)$$

T_c is directly proportional to the energy gap $\Delta\left(T\right)$ [65]:

$$\Delta T \approx 3.06kT_c(1-\frac{T}{T_c})^{1/2} \quad (4)$$

If various quantities in formula (2) are the same, then $T_c \propto \hbar(\mathrm{H})$. We suppose that various quantized phenomena, whose part at least, can be described by the extensive quantum theory [56] $\hbar \to H$, and T_c can be greatly improved. The density matrix allows us to define the macroscopic wave function of a suitable effective single particle.

Usual superconductivity is based on Cooper pair. But, the electron pairs are mutually exclusive, it must be formed reverse magnetic moments attract each other. Attraction is greater than repulsion under certain conditions. On the one hand, the extensive Cooper pairs try to explain high temperature superconductivity. On the other hand, Cooper pair is extended for spin J=0 system, which be probably related to PEP violation [66-70], etc.

For superconductivity Ginzburg and Landau (GL) postulated the existence of an order parameter denoted by ψ, Ginzburg-Landau (Abrikosov) equation is [2]:

$$-\frac{\hbar^2}{2m^*}\nabla^2\psi + a\psi + b\psi\left|\psi^2\right| = 0. \quad (5)$$

Here $\alpha = a\left(T - T_c\right)$. The new parameter m* plays the role of an effective mass for the quantum system with macroscopic wave function. We may assume $\frac{\hbar^2}{2m^*} \to \frac{(\hbar')^2}{2m}$, so $\hbar$ is changeable. This will be the simplest method of high-T_c superconductivity.

Let $\xi\left(T\right) = \frac{\hbar}{\sqrt{2m\alpha}}$, Equation(5) may be simplified to:

$$-\xi^2\left(T\right) = \frac{d^2f}{dz^2} - f + f^3. \quad (6)$$

Equations (1)(2) are all nonlinear equations. The solution of Equation(6) is [71]:

$$f\left(z\right) = \tanh\frac{z}{\sqrt{2\xi}}. \quad (7)$$

It is similar form with soliton.
Equation (5) is the time-independent nonlinear Schrödinger equation, whose soliton is invariable that may explain superconductivity [72].
It is known that the nonlinear Schrödinger equation

$$\phi_{xx} + i\phi_t + k\left|\phi\right|^2\phi = 0, \quad (8)$$

has a soliton solution [72]:

$$\phi_s = \phi_0 \sec h[\frac{k}{2}\phi_0\left(x - \mu_e t\right)\exp[i(\frac{\mu_e}{2})(x - \mu_c t)],$$

where the variable $\eta = x - \mu_e t$. Let

$$\phi = \exp[i\frac{\mu_e}{2}(x - \mu_c t)]v,$$

the equation (8) may become

$$\frac{dv}{d\eta} = [C + av^2 - \frac{k}{2}v^4]^{1/2},$$

where $a = (\mu_e / 2)^2 - (\mu_e\mu_c / 2)$. When C=0, the soliton solution is

$$v = \sqrt{\frac{2a}{k}}\sec h^2\eta.$$

Soliton has an unchanged shape and speed in motion with energy conservation, so it may describe superconductivity [72].

Further, we should derive various solutions of Schrödinger equation with potential V (the Coulomb potential, magnetic moment potential, phenomenal potential and so on).

Usual mechanism of superconductivity is the Cooper electron pair and BCS theory. A model of the high-T_c superconductivity is possibly bag model.

In 1987 Weinstein applied the SLAC bag model [73] to study the high temperature superconductivity phenomena, and the obtained voltage number and its ratio agree with experiments [74]. This suggests a new study approach for the superconductivity theory.

Schrieffer, et al., emphasized short-range magnetic correlation, and proposed the spin-bag mechanism of high temperature superconductivity, whose quasiparticle has the traditional quantum number of holes in solid physics, i.e., charge +e and spin 1/2, but has the strong dressing effects [75]. The excitation of spin-bag may lead to a pseudogap in the electronic spectrum, and calculated the self-energy of spin-bags and their pairing interaction, which are consistent with the corresponding results in the weakly doped ordered antiferromagnet [76].

high-T_c superconductors are all composite materials with complex structures. The basic elements of superconductor have complex entanglement relations, and form bag, and coupled pairs. Based on the bag model in particle physics applied widely, and its associated Friedberg and T.D. Lee nontopological soliton theory of fermi fields [77,78], the bag model of high temperature superconductivity theory is obtained. At this case, superconductivity can be generalized from Cooper pair to fermion pairs, bags, etc., as a possible theoretical interpretation. If the bag contains multiple electrons, the electrons corresponding to the high temperature superconductor are originally tightly bound together. Multiple electrons of strong coupling into bags also makes the critical temperature of superconductivity should be higher.

The bag model is a mechanism of confinement quarks, and its similarity to superconductivity has been discussed by T.D. Lee, et al. The σ field-quark interaction in the generalized hadronic bag model corresponds to the phonon-electron interaction in the BCS theory. The bag corresponds to the Fermi sphere, where the distribution function is the Fermi-Dirac (FD) distribution. The equation is nonlinear in this case. The soliton solution can include the changeable curve of negative susceptibility of superconductivity with temperature and the FD distribution function of the Fermi sphere. Results are similar to FD distributions obtained from MIT bags especially for the limiting n→0.

Assume that the high temperature superconductivity corresponds to the SLAC bag, it corresponds to the nonlinear equations:

$$\frac{du}{d\rho}=(-1+u^2-v^2)v\,, (9)$$

$$\frac{dv}{d\rho}+\frac{2v}{\rho}=\left(1+u^2-v^2\right)u\,. (10)$$

The nonlinear equations can be applied by the qualitative analysis theory, and explore its meaning. The general nonlinear equations should have soliton solutions, chaos solutions, and correspond to the fractal and the scaling of time.

MIT bag is limit of n→0, above equations are linear, and have not the nonlinear terms. SLAC bag is limit of n→∞:

$$\frac{dx}{d\tau}=yx\,,\ \frac{dy}{d\tau}=-\tau x\,. (11)$$

Here x = uv, $y=(u^2-v^2)$, $\tau=2(\rho\text{-}1)$. Equations may become to:

$$x''=yx'-\tau x^2=xy^2\,,\ y''=-\tau x'=-\tau xy\,. (12)$$

This may be that Cooper pair has developed into a bag-bound state of many (2N) particles. Bag corresponds to nonlinear sigma models. It may be related to energy, symmetry, etc., and to the extensive quantum theory [55-57]. The both developments may form multiple layers, structure change, and different phase change points. Meanwhile, different quantum constants will have an influence on the BCS theory, e. g. $T_c \propto \hbar$ (H).

Under certain external and internal conditions, all particles pairs each other and "condense" into a single state, and form a highly ordered, long-range coherent state. Because of the pair, it is the Bose-Einstein momentum condensation. At this case, all particles behave almost exactly the same, and the whole motion of a large number of particles is the same as that of one of them, and the whole particle system can be described by a macroscopic wave function and the corresponding equation. At very low energy, fermions condense as quasi-bosons, and PEP is not possibly applicable [66-70]. Further, we can also apply some theories related to the bag model, and a generalization of quantum chromodynamics (QCD). Bag may be developed to three dimensions from one dimensional string and two dimensional brane.

Moreover, low temperature and high temperature superconductivity may correspond to light and heavy fermion systems, respectively, so that the superconductivity temperature is proportional to a certain mass. At low temperature, electrons constitute Cooper pair; at high temperature, some heavy fermions are Pauli includible, multiple electrons can form bag. While the high and low temperatures are relative. Probably, low temperature superconductivity is boson (fermion pair); high temperature superconductivity is anyone.

High-temperature superconductivity theory is mainly a strongly correlated quantum theory. Many experts believe that the high-temperature superconductivity mechanism must establish a new many-body quantum theory system. The extensive quantum theory is a method to study many-body quantum theory. The superconductivity mechanism of iron-based superconductor may be magnetic related, and may theoretically include spin fluctuations or inter-orbital electron pair transitions. Now the characteristic energy scale is much larger than the average phonon energy scale in the BCS theory, so it is possibly to derive a higher superconductivity transition temperature. High-temperature superconductivity is related to the magnetic flux dynamics, so it should be easier under some magnetic field [43,45].

The relation between high temperature superconductivity and spin should be researched. Electrons long-range correlation can achieve the order of nm.

Equations (5) and (8) should be generalized to high-temperature superconductivity, such as adding the nonlinear term $\psi^5, ...\psi^n$, or changing its form. It can be taken as a generic form $f_{NL}(\psi)$, and search conditions of the critical temperature and high temperature superconductivity.

For various Josephson effects the no-voltage-DC superconductivity current formula is:

$$J_s = \frac{\hbar a e_s}{m_s} n_s \sin\Phi = J_0 \sin\Phi \text{ , (13)}$$

The alternating current formula is:

$$J_0^{\omega} = J_0 \sin(\Phi - \frac{e_s V}{\hbar} t) \text{. (14)}$$

If they generalize to the extensive quantum theory, where H changes large and J_0 becomes large, the same results can be obtained from similar equations. If m_s also becomes larger, then J_0 can be the same results.

Superfluidity and BEC

The superfluid includes the Bose liquid ^4He and the Fermi liquid ^3He. ^3He has a nuclear spin J=1/2, and a mass m=5.01×10^{-24}g, and the effective mass m*=2.8m and m=5.5m at the pressure P=0 and P=30 bar [79].

^3He as fermion and ^4He as boson all have superfluidity, which shows that it is independent of statistics, and is probably related to unified statistics [66]. But, both superfluidity temperatures are different. ^3He has two superfluidity phases, which is similar to Cooper pair. Superfluidity must be understood by the generalization of BCS theory describing superconductor. Among them, atoms replace electrons to form Cooper pair, and their attractive force regulation mechanism is replaced by spin fluctuation instead of phonons. They are the thermomechanical effect (fountain effect) and S=0, etc.

Recently, Chen, et al. searched generating giant vortex in a fermi superfluid via spin-orbital angular-momentum coupling [80]. Luo, et al. studied spin-twisted optical lattices [81]. Long, et al. discussed spin susceptibility above the superfluid onset in ultracold Fermi gases [82]. Liu, et al. researched universal dynamical scaling of quasi-two-dimensional vortices in a strongly interacting fermionic superfluid [83].

Grémaud, et al. discussed pairing and pair superfluid density in one-dimensional two species fermionic and bosonic Hubbard models [84]. Kanai, et al. studied true mechanism of spontaneous order from turbulence in two-dimensional superfluid manifolds [85]. Kim, et al. discussed critical energy dissipation in a binary superfluid gas by a moving magnetic obstacle [86]. Choi, et al. searched spatially modulated superfluid state in two-dimensional 4He films [87]. Stockdale, et al. studied dynamical mechanisms of vortex pinning in superfluid thin films [88].

In 2022 Braidotti, et al. measured Penrose Superradiance in a photon superfluid [89]. Biss, et al. used ultracold Fermi gases with tunable interactions realized the BEC-BCS crossover from a Bose-Einstein condensate (BEC) of molecules to a Bardeen-Cooper-Schrieffer (BCS) superfluid of weakly bound Cooper pairs, and measured the full momentum-resolved low-energy excitation spectrum of strongly interacting ultracold Fermi gases, and calculation better agreement with data [90]. Yu, et al. found the exact propagating topological solitons in the easy-plane phase of ferromagnetic spin-1 Bose-Einstein condensates, manifesting themselves as kinks in the transverse magnetization. Solitons have two types: a low energy branch with positive inertial mass and a higher energy branch with negative inertial mass. Both types become identical at the maximum speed, at which the soliton undergoes oscillations caused by both transitions [91].

Applications of superfluidity include the dilution refrigerator, high-precision instruments, etc. Further, physicists found the superfluidity gas, which is 50 nK lithium-6. In 2021 Ferlaino F, et al. found supersolid [92], which is solid with superfluidity.

For superfluidity and BEC the Gross-Pitaevskii equation is:

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi-\lambda\psi+g|\psi|^2\psi. \quad (15)$$

This is also the nonlinear Schrödinger equation. For Equation(15) in superfluid, assume that h→H=h/m, so k=hn/m=Hn. These are related to the Bose-Einstein condensation (BEC).

In Landau theory the quasiparticles are the elementary excitations of the system, which as effective particles like the phonons in a solid or the spin waves in a magnet. It is namely a type of soliton. The Ginzburg-Landau theory is macroscopic. The concept of the macroscopic wave function is central to understanding atomic Bose-Einstein condensates (BEC), superfluid and even superconductivity. The macroscopic wave function, and its phase, is not a true wave function in the sense of elementary quantum mechanics. In particular it does not obey the fundamental principle of superposition, and one cannot apply the usual quantum theory of measurement or Copenhagen interpretation to it. The macroscopic wave function behaves much more like a thermodynamic variable [2]. The macroscopic wave function arises naturally from the physics of coherent states, which can be defined for fermions as well as for bosons, and can be fermion pairs.

The fundamental characteristics of soliton and chaos in nonlinear equation are completely different. All nonlinear equations with a soliton solution may derive chaos. While only some equations with a chaos solution have a soliton. The conditions of the two solutions are different. When some parameters are certain constants, the soliton is derived; while these parameters vary in a certain region, the bifurcation-chaos appears. We researched the double solutions with soliton and chaos in some nonlinear equations [93,94].

From Equation(5) let $v=\sqrt{2a/k}\sin x$, the equation is

$$x'=\sqrt{a}\sin x, \quad (16)$$

which has the chaos solution. For a stable state whose energy is H, if *k=-b<0*, the equation will be

$$\phi''+H\phi-b\phi^3=0, \quad (17)$$

whose integral is

$$\phi'=(C-H\phi^2+\frac{b}{2}\phi^4)^{1/2}. \quad (18)$$

Let $C=H^2/2b$, so

$$\phi'=\sqrt{\frac{b}{2}(\frac{H}{b}-\phi^2)}. \quad (19)$$

When $|\phi|<\sqrt{H/b}$,

$$\phi_s=\sqrt{\frac{H}{b}}th(\sqrt{\frac{b}{2}}\eta+C_0). \quad (20)$$

It is the simplest soliton with a bell shape. Using a substitution $\phi=Hx/\sqrt{2b}$ for Equation (19), and it became a difference equation

$$X_{n+1}=1-\frac{H}{2}X_n^2. \quad (21)$$

It is an equation, which has the chaos solution, and its parameter determined the bifurcation-chaos is $\mu=H/2$. It is known that soliton may correspond to superconductivity, and chaos corresponds to phase transformation and to probably superfluidity and BEC, where critical parameters correspond to the critical temperature.

In superconductivity Cooper pair of BCS theory is two times of the electron. This and the superfluidity are all low temperature macroscopic whole quantum effect, and both are microscopic correspondence. They are all Bose-Einstein (BE) statistics.

The unified theory of superfluidity and superconductivity may correspond to the gauge symmetry breaking.

Some Predictions and Discussion

Superfluidity, superconductivity and BEC are all some macroscopic group effects, and should have common something, and various corresponding low temperature effects, such as the analogous capillarity. They are related to the laser light. We predict that the high-T_c superfluidity and BEC should exist. For example, the temperature of the superfluid in the neutron star may reach to 10^8K.

They probably must be the mixture of the multivariate and multiple components. The mechanism and theory of superconductivity and superfluidity are similar. After the theoretical generalization of the former, it should be able to predict the transition temperature of the latter, so we may research their theories and experimental predictions. In addition, there is the laser after the conditional expansion. BEC as an anyon consisting of e-e interactions, and PEP breaks. There may also be a high-temperature quantum Hall effect. Various aspects of macroscopic states can be compared to each other.

Cooper pair derives usual superconductivity. This extends to general fermion pair, whose stabilization will form the excited fermions, and fermion condensation

(FDC). For nucleons neutron pair $n\overline{n}$ is uncharged and is non-conductor. It is possible that at low temperature the proton pair $p\overline{p}$ combines by strong interactions, and even multiple protons form nuclei, bag, etc. This is the nuclear, bag superconductivity. It develops into the quantum theory.

Stable photons may form laser, and other excited bosons are unstable. Stable neutrino pair $\nu\overline{\nu}$ corresponds to photon in the de Broglie photon melting model. pp, pn and nn are already nuclei, whose conditions are also not necessarily low temperatures, and have led to new properties such as strong interactions. Other some particles are unstable. Further, it can be the condensation of various atoms and molecules, which are divided into bosonic type, or fermionic pair type.

The anomalous is that the spin dots in the solid state ^3He at low temperature have higher entropy than the liquid state, and the entropy difference $\Delta S = S_l - S_s$ is positive at high temperature, $\Delta S = 0$ about T=0.3K, and ΔS is negative at below 0.3K [71].

We predict that superconductivity may have similar entropy decrease, S=0 and fountain effect, etc., in superfluidity. But both generally cannot be isolated systems.

If superconductivity, superfluidity and BEC are nonlinear, so they should have some nonlinear effects. Further, assume that they may be described by nonlinear equations and nonlinear wave, which has various self-focusing, and self-modulation, etc [95]. When the amplitude of nonlinear wave attains a maximum at the centre of the light beam, the wave focuses on its centre, i.e., the self-focusing effect. And its occurrence exists a threshold value of the amplitude [95]. The nonlinear Schrödinger equation may develop to the nonlinear sine-Gordon equation and self-induced transparency, and the nonlinear Klein-Gordon equation, etc. Probably, superconductivity, superfluidity and BEC have similar effects.

We can assume that they are entangled, and establish the corresponding phenomenal theory.

In a word, various macroscopic quantum phenomena are observed and are applied widely, and corresponding theories should be researched.

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