

## Is the Lottery Paradox Psychologically Realistic?

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### Abstract

The lottery paradox is treated as a serious epistemological problem. Philosophers try to solve it by modifying their concepts of rationality. I argue that this is unnecessary. The paradox never occurs in reality, since a crucial assumption -- that people accept that certain lottery tickets will not win -- is refuted by cognitive science. We always exaggerate small probabilities, such as the possibility to win a lottery. Of course, at some point we lose faith in the chance. However, this lack of confidence is not based on a mature consideration of the ticket's chances -- but a result of an exchange of questions, our desperate last attempt to form an answer. Hence the lottery paradox is a purely theoretical problem, parted from reality.

### 1

The famous lottery paradox was formulated by Henry Kyburg<sup>1</sup>. It rests on three principles of rational acceptance:

- i. If it is likely that p, then it is rational to accept that p.
- ii. If it is rational to accept that p and it is rational to accept that q, then it is rational to accept that (p and q).
- iii. It is not rational to accept that (p and not-p).

These principles all seem plausible. But imagine a fair lottery with a thousand tickets. Since the rules say so, it is likely -- and thus rational to accept, by (i) -- that one of the tickets will win. However, the chance that the first one will is only one permille, so it is rational to accept that it will lose. Likewise, it is rational to accept the loss of the second ticket. Actually, for each of the thousand lottery tickets, it is rational to accept that it will lose. All of these statements follow by (i). By (ii) we may add them all together. It is thus rational to accept that the first ticket will not win, the second ticket will not win and so on for

every participating ticket. That is, (i) and (ii) imply that it is rational to accept that no ticket will win and that at least one of them will. However, (iii) tells us that it is not rational. This is the contradiction called the lottery paradox.

Broadly speaking, there are three types of responses, each of which focuses on either of the rules (i), (ii) or (iii). Philosophers have dealt with (iii)<sup>2</sup> or (ii)<sup>3</sup>, but the common way to block the lottery paradox is to modify (i)<sup>4</sup>. However, "[a]lthough there is a consensus... that one should deny that it is rational to accept that a ticket of the lottery loses, there is less agreement over why this should be so"<sup>5</sup>.

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<sup>1</sup> (Kyburg, 1961)

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<sup>2</sup> (Klein, 1985)

<sup>3</sup> (Kyburg, 1970)

<sup>4</sup> (Levi, 1967), (Cohen, 1988), (Lewis, 1996)

<sup>5</sup> (Wheeler, 2007)

Two conclusions are to be drawn. First, philosophers think "the lottery paradox poses serious challenges to our assumptions about knowledge and rational belief"<sup>6</sup>. Second, they say the problem concerns what is rational to accept. When the lottery paradox establishes that there is a contradiction in the concept of rational acceptance, it is, they say, not the acceptance part that is malfunctioning, but the rationality condition. No one questions the assumption that people from time to time accept that certain lottery tickets will lose. This has been the outset.

In this paper, I argue that the lottery paradox may not be as important a problem as philosophers have treated it. I do this not by noticing that it is irrational to accept that certain lottery tickets will lose. Rather, I observe that such propositions do not seem to be accepted in the first place. In section 2 I translate "no one believe that a certain lottery ticket will lose" into terms viable in cognitive science and philosophize on how to analyze results from questionnaires. Dual process theory, which is the framework I will use, is introduced in section 3. Thereafter, my investigation begins. In section 4 I argue that human mind always exaggerates small probabilities, such as a lottery ticket's chance to win the first prize. In section 5 I admit that human mind indeed loses faith in probabilities when they grow sufficiently small. However, I notice that this is done by dropping confidence suddenly, in a discontinuous way. This, I claim in section 6, is because human mind considers an inadequate question. Such behavior, I argue, calls for disqualification from the investigation. Hence, I conclude, human mind does not really accept that a certain lottery ticket will lose after all, and the lottery paradox turns out to be psychologically unrealistic. Conclusively, I argue that problems which do not correspond to reality are less important than problems which do. In section 7 I summarize my findings.

My thesis is -- sloppily expressed -- that people actually do not accept that certain lottery tickets will lose. This is, of course, a philosophical claim. In order to challenge it in cognitive science, I have to modify it. In particular, I have to express it as a question. Let us say I have a ticket in my hand and ask a thousand men on the street:

(1) Do you think this lottery ticket will win?

If -- sloppily expressed -- anyone of them answers "no," my thesis is falsified. But neither the question nor the answer is of course good enough for my investigation.

They are both overly vague. I must fix the probability, which is easily done. I fix the value of the winning prize, which is easily done. I must define "do you think the ticket will win?" This may be done by requesting the participant to evaluate the lottery ticket. That is, instead of "do you think it will win?" I ask "how much will you pay for it?" The exact question I ask lottery participants is thus:

(2) How much will you pay for the chance  $x$  to win  $y$ ?

My exact claim is that no lottery participant will ever value his chance  $x$  of winning  $y$  to "0," given that both  $x$  and  $y$  are positive. Here, I will not perform a study in cognitive science. Rather, I will examine previous experiments, in order to establish what people would answer to (2).

Before starting, let me underline a self-evident yet crucial point regarding questions: I expect answers which correspond to my question. Those answers which correspond to other questions I disqualify from my investigation. Imagine I have an object and wonder how tall it is, and thus produce a small survey. On purpose I do not ask: "Do you think this object is big?" for I see the question is vague and thus would generate a bunch of ridiculous answers. Rather, I ask: "Do you think this object is big compared to a normal teddybear?" Answers to this question offer me a comparison, and hence valuable information. For I expect the respondents to pay respect to my defined question. That is, I expect them to consider the size of the object, compared to the size of a normal teddy bear. If they do not, if they at random compare the object to the moon, their answers will be like spam to my investigation. Their answers are as valuable to me as those offered by survey participants who do not understand the language on the sheet. Can I establish such a failure, I had better disregard from their contribution to my study.

In the same manner, I want the participants of my thought survey to consider the question (2). That is, I want the answers to be on the form: "I see that a ticket offers me a chance  $x$  to win  $y$ , and after mature consideration I will pay such and such for it." If I can establish that part of them diverge from this pattern: if they consider (1) instead, I may disregard from their answers.

### 3

My framework will be dual process theory. Given this view, human cognition consists in two fundamentally

<sup>6</sup> (Nelkin, 2000, p. 408)

different states of thinking<sup>7</sup>. Following Daniel Kahneman in his book *Thinking, Fast and Slow*<sup>8</sup> I call the two states System 1 and System 2.

System 1 is the automatic, unconscious thinking. When I feel the urge for chocolate and go to the kitchen, System 1 controls my actions. When seeing an angry face, System 1 tells me to retreat. When you read these sentences, System 1 processes the letters of the roman alphabet. The introductory picture that usually<sup>9</sup> preface dual-process theory papers, describes System 1 as evolutionary old, constant amongst humans, and shared with the animals. System 2 on the other hand, is evolutionary new, variable amongst human beings, and not shared with animals. For System 2 does conscious thinking. When I decide that the chocolate will have to wait till evening, System 2 remembers my New Year's resolution. When explaining to myself why a person looks angry, System 2 is working. If you have never got in touch with dual process theory before, System 2 tries to understand the content of this text for you. Kahneman points out the differences of the two systems:

Conscious doubt is not in the repertoire of System 1; it requires maintaining incompatible interpretations in mind at the same time, which demands mental effort. Uncertainty and doubt is the domain of System 2<sup>10</sup>.

System 2 is a rule-based state of mind, which tries to describe the world with logical structures<sup>11</sup>. Mathematical rules are perfect examples, and probabilistic ones in particular. That is, if a random variable has a certain distribution function, we can derive the expected value and the standard deviation. If we know that a lottery ticket has a certain chance of winning, and we know the value of the prize, we may compute the expected value of the draw. This we do with the aid of System 2. What we cannot derive by such rules is what actually is going to happen. For System 2 is always aware of the risk that the outcome will not be as expected. By contrast, System 1 is not rule-based but associative and dependent on statistical data to draw conclusions. If all dogs I have seen

in my life have been barking and biting, I may draw the conclusion that all dogs are like that<sup>12</sup>.

If a human being is to consider the question (2) and answers "0," she does this with the aid of either System 1 or System 2. By the discussion above, I repudiate the latter option. If my thesis is to be rejected, it must be so by System 1. Thus, I will focus on System 1 for the rest of the article.

#### 4

The first thing to note about System 1, is that it overestimates small probabilities. This exaggeration is spelled out well by Kahneman, and following him, I will call it the possibility effect<sup>13</sup>.

Experimental studies indicate a possibility effect. In one of them, participants were asked to rank the probability of lethal events<sup>14</sup>. That is, they read a way of dying, and were supposed to answer how probable they thought the event was. Probable lethal events, such as heart disease or cancer, were highly underestimated. On the other hand, unlikely events were overestimated. The less likely event, the more did the participants overrate it. This is a list of the ten least probable events examined in the study, and how the risks that they occurred were evaluated<sup>15</sup>:

Lethal event	The actual rate	The judged rate
Tornado	90	688
Venomous bite or sting	48	535
Polio	17	202
Whooping cough	15	171
Smallpox vaccination	8	38
Fireworks	6	331
Measles	5	331
Botulism	2	379
Poisoning by vitamins	1	237
Smallpox	0	88

**Table 1:** Least probable events.

The study is clearly in line with the possibility effect. The probabilities that the unlikely events will occur are grossly exaggerated. I claim that the same goes for lottery

<sup>7</sup> There are other ways of interpreting the human mind, see for instance (Cleeremans, 2002). However, in this article I will follow the much-appreciated dual process theory, and see what conclusions may be drawn in that framework.

<sup>8</sup> (Kahnemann, 2011)

<sup>9</sup> (Evans, 2009), (Carruthers, 2009), (Samuels, 2009)

<sup>10</sup> (Kahnemann, 2011, p. 80)

<sup>11</sup> (Smith, 2000)

<sup>12</sup> (Sloman, 1996)

<sup>13</sup> (Kahnemann, 2011, pp. 310-321)

<sup>14</sup> (Lichtenstein, 1978)

<sup>15</sup> The original paper is from 1978, which may explain the fear of diseases like smallpox and polio. The rates are specified per 205,000,000.

participants. When holding a lottery ticket in your hand you will exaggerate your chances to win.

## 5

But we do not care about every small probability. A meteor may crush into my head, but the risk is so faint, that I ignore the risk. In the same manner, I neglect some improbable opportunities I may have. And, seemingly devastating for my case, I may even drop confidence in my lottery ticket's chance to win. That is, in spite of the probability effect there is a decline of belief when the probability decreases. But this decline of belief is not continuous and smooth -- rather, it is sharp and discontinuous. It is like System 1 suddenly has lost sight of the chance. Or, as I prefer to call it, System 1 has passed the threshold of perceived possibility. Kahneman writes:

It is difficult to assign a unique decision weight to very rare events, because they are sometimes ignored altogether, effectively assigned a decision weight of zero. On the other hand, when you do not ignore the very rare events, you will most certainly overweight them<sup>16</sup>.

In a study which concurs with the idea of a threshold of perceived possibility, 64 participants attended. They were given ten dollars each, gained one dollar each time a white ball was drawn, lost four dollars each time a red ball was drawn but could be saved from loss if they had bought an insurance<sup>17</sup>. The participants were to bid for insurances at auctions. It would be rational to place a bid which was in accordance with the risk to draw a red ball. That is, if there is a ten percent risk of losing ten dollars, simple math tells us that a reasonable price of insurance is one dollar. But, as we now know, the possibility effect tricks System 1 to exaggerate small probabilities.

When the probability to draw a red ball was twenty percent, nearly half of the participants bid what was reasonable. When the probability was ten percent, considerable possibility effects showed up. Additionally, ten percent of the participants chose to be lucky -- they ignored the risk of the red ball occurring. But when the probability of drawing a red ball grew fainter, the effects were even more significant. Here are the results which were collected when only one of a hundred balls was red<sup>18</sup>:

Percent of the participants	Bid divided by expected value
26	0
5	0.3
4	0.5
18	1
27	2
12	4
6	9
2	18

We see that only eighteen percent of the participants bid what was reasonable. Else, the group was starting to divide into two camps. One that overestimated the risk of the red ball occurring -- forty-seven percent of the participants did this, and another that neglected the risk totally -- twenty-six percent of the participants chose that path. The possibility effect on the one hand and the threshold of perceived possibility on the other are show-offs indeed.

The conclusion is that System 1, when facing a small probability, has a decision to make: either it pays attention to the small chance to win, and then exaggerates the probability, or it ignores the chance altogether. When the actual probability is reasonably high, most human beings will go for the former option; when the actual probability grows faint, many will choose the latter path instead. This experiment and this analysis, I claim, correspond to the situation when lottery participants are asked the question (2).

Two questions arise. First, how can I claim that the lottery paradox is not psychologically realistic when presenting results which show that people indeed may believe in loss? Second, throughout the investigation, the human mind has proven unable to understand probabilities properly, and it has been worse the smaller the probability. More exactly, it has exaggerated probabilities more the smaller the probability. How come it suddenly neglects the chance altogether? Actually, we will see that the answer to the second question will pose a satisfactory explanation to the first one as well.

## 6

System 1 generates answers to all questions. Kahneman notices:

A remarkable aspect of your mental life is that you are rarely stumped. True, you occasionally face a question such as  $17 \times 24 = ?$  to which no answer comes immediately to mind, but these dumbfounded moments are rare. The normal state of mind is that you have intuitive feelings and opinions about almost everything

<sup>16</sup> (Kahnemann, 2011, pp. 315-316)

<sup>17</sup> (McClelland, 1993)

<sup>18</sup> In the original paper the percentage rates are only presented in a diagram. Here, the bars are replaced by numbers, carefully read by the author. However, minor mistakes could still have been made.

that comes your way. You like or dislike people long before you know much about them; you trust or distrust strangers without knowing why; you feel that an enterprise is bound to succeed without analyzing it. Whether you state them or not, you often have answers to questions that you do not completely understand, relying on evidence that you can neither explain nor defend.<sup>19</sup>

This urge to find answers to any question is similar to another feature of System 1: it is immune to ambiguities. Kahneman exemplifies with the sentence: "Ann approached the bank," which could mean that a woman walks towards a river, but may as well insinuate that a girl is going to lend some money. However ambiguous a phrase, System 1 will choose one of the reading options<sup>20</sup>. It will never stick to the humble "well, the information could mean this, but it is not necessary -- it could as well mean that."

These properties of System 1 have very interesting implications: it will do what it takes to find answers to questions. In a study, participants were gathered in front of a rigged wheel of fortune<sup>21</sup>. Only the numbers 10 and 65 occurred, so each attendant saw either 10 or 65. Afterward, he or she was asked the seemingly irrelevant question:

"What is your best guess of the percentage of African nations in the UN?"

The participants which had seen number 10 on the fortune wheel formed one group of respondents, and the ones which had faced number 65 formed another. As it turned out, the average answer in the first group was 25, while the average answer in the second group was 45. The conclusion is that System 1 is so eager to produce answers, that it uses irrelevant information in the process, affecting the outcome.

On the other way around, experiments have shown that a slight exchange of questions result in different answers. In another study, participants were asked how satisfied they were with their lives<sup>22</sup>. Two groups were put in front of the same questions. The first group's answering sheet read:

"How are things going with your dating life?" and only then,

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<sup>19</sup> (Kahnemann, 2011, p. 97)

<sup>20</sup> (Kahnemann, 2011, p. 80)

<sup>21</sup> (Kahnemann, 2011, p. 119)

<sup>22</sup> (Strack, 1988)

"How is your life going in general?"

That is, the survey participants were lured to think about their dating life, when evaluating their life in general. The result thus showed a remarkable correlation between dating life ranking and general life ranking. The second group read the same question, but in opposite order. In their answers, the correlation was absent.

**Kahneman explains:** in order to maintain its habit to always give an answer, System 1 prefer a simple heuristic to find the answer to a question. Thus it does not always consider the hard, complex question. Instead, it replaces it with an easier one. Suppose someone asks me:

"How happy are you these days?"

This question is rather complicated, and to be able to give an answer quickly, I may replace the difficult question by the less complicated:

"Are you happy right now?"

When thinking: "yes, I'm fine right now," I automatically form the belief that I am rather happy these days<sup>23</sup>.

In most cases, these simple heuristics cannot be tracked. However, in the wheel of fortune-study and the dating satisfaction-experiment the participants' answers clearly showed bimodal results. The participants which had considered one question answered one way, the ones which had considered another question answered differently. Thus, I propose, in a large group of people, where a bimodal response to a question is apparent, we may suspect that two different heuristics have been used.

What is interesting to note, is that a similar bimodal response showed up in the insurance auction, and would most certainly occur if a survey was done on peoples' confidence in their tickets' chances at a lottery. When probabilities grow tiny, people either overestimate them or neglect them. We know the reason why they exaggerate them: it is due to the possibility effect. What happens when they suddenly neglect the probability altogether, I propose, is that they consider an easier question. Let us have another look at the original question (2):

"How much will you pay for the chance x to win y?"

It demands a rather complicated evaluation. System 1 will be tempted to replace it by:

"Have you ever won on a lottery?" or,

"Do you even think that you can win?"

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<sup>23</sup> (Kahnemann, 2011, pp. 97-105)



To our purposes, it is not important to find out exactly which question System 1 tends to use as a substitute. What is clear, is that the optional answers are "yes" or "no." If positive, the respondent must consider the complicated question (2) again. He must evaluate the chance of the lottery ticket to win. In accordance with the possibility effect he will exaggerate it. Else, the answer is negative. Then, it must somehow be translated into a meaningful answer to the original question (2). Since the agent is supposed to assign a value, he chooses: "0." This is the mechanics behind the huge leap between the possibility effect and the threshold of perceived possibility.

So how can I claim that the lottery paradox is not psychologically realistic when presenting results which show that people indeed may believe in loss? Recall my discussion from section 2. When handing out a survey, I expect the participants to consider the questions on the sheet. The questions are deliberately developed for my investigation's purposes. I want the participants to evaluate how much a lottery ticket is worth, and to get an as accurate result as possible, I hand out a large number of sheets. However, if some of the participants consider other questions -- easier questions -- my study is not improved at all. Thus I am only interested in the sheets from the participants which understood the questions, as most psychologists are intrigued only by those who know the language on the sheet. If there is a mark in the box: "I do not know English," the answers to the following questions should be ignored in the study. In the same manner, if I can identify the participants who did not consider the question (2) I had better disregard from their answering sheets.

And actually, I am able to identify those participants. They are the ones which have set the value of the lottery ticket to "0." That this is not a proper evaluation is seen by the bimodal result: one group has exaggerated the chance of the ticket, the other has neglected the chance altogether. The latter group has, also, considered a question distinct from (2). One may object that some of the disbelievers may have done their evaluations in a proper manner. Pointing at experiments where humans have considered different questions and have reached a similar bimodal result, I claim that such conclusion is implausible. The bimodal distribution of answers clearly shows that two different questions have been considered. Therefore, I disregard from the disbelievers' answers and conclude that no one really evaluates a certain lottery ticket's chance to win the prize to "0." Hence the lottery paradox is undermined -- it is not psychologically realistic.

What are the implications of my thought experiment of cognitive science? Well, there are problems which are important, and there are ones that we had better not care about.

Suppose my doctor warns me: "you should not spend time in the sun during summer, if the US is situated in Europe." Imagine I react thus: I do not worry as it is spring-time right now -- but when summer arrives I will stay inside in accordance with my doctor's advice. However, an important problem has been posed to me. Exactly when, I ask, does summer start? This is a crucial question, for if I spend time outside during summer, terrible consequences will presumably occur. Only hours later I recall my doctor's additional conditional -- his cautioning only holds if the US is situated in Europe. This is false, and hence my doctor's cautioning is as well. The question when summer starts is still not trivial. However, the issue is irrelevant, for the effects will never be of practical import.

The same, I claim, goes for the Lottery Paradox. In this article I have tried to show that the paradox will never occur in reality. And if it does not occur in reality, it is not crucial for us to solve it.

## 7

I have dealt with the lottery paradox. Although a philosophical problem, I have made an approach from a cognitive scientist's point of view. For the importance of the lottery paradox rests on the assumption that people actually believe that certain lottery tickets will lose. This is the claim I have tried to refute in this paper. I have noticed that human mind always exaggerates small probabilities. Thus far, it seemed I had a case -- people tend to believe in lottery tickets' chances. But I encountered an obstacle: when the probabilities grow sufficiently small, people suddenly drop confidence in them. The lottery paradox appeared to be psychologically realistic after all. But why do people suddenly neglect small probabilities? I have argued it is because they use simple heuristics. Instead of considering what a certain lottery ticket is worth, they dogmatically ignore its chance to win. That is, they do not give an answer to the relevant question. Thus, I have claimed, we had better disregard from their answers, as we disregard from any answer which is produced at random. Hence the foundation of the lottery paradox breaks down. It is not psychologically realistic. And herewith I urge philosophers to spend their time on something else.

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