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Copernican Multilateral Revolution: Gradual Reconciliation of Aristotelean Terrestrial Physics and Ptolemaic Celestial Mathematics

Nugayev RM*
Volga Region State University, Russian Federation

***Corresponding author:** Rinat M Nugayev, Volga Region State University, Kazan 420138, Universiade Village, Republic of Tatarstan, Russian Federation, Email: rinatnugaev@mail.ru

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Abstract

Copernican Revolution is elicited in the distinctive context of intense interaction of Aristotelean and Ptolemaic subtle theoretical languages. It is unfolded that already within the Ptolemaic research program the mathematical exactness increasingly deviated from the blunt tenets of Aristotelean qualitative physics. Aristotelian - Ptolemaic pagan cosmology could not help but be exposed to repeated attacks during the Middle Ages since it apparently confronted the principles of monotheism not admitting the stiff and impenetrable demarcation line between the celestial and mundane realms. All different worlds should have one and the same Creator. Starting the unification, Copernicus in effect paved the way for Galileo's and Newton's mathematical physics.

Keywords: Aristotelian Qualitative Physics; Ptolemaic Mathematical Astronomy; Monotheistic Weltanschauung; Copernicus; Galileo; Kepler; Descartes; Newton

A Terse Introduction: Copernican Vs. Ptolemaic Diverse Research Programs

Why did bold and abstruse Copernican research program squeeze out refined and entrenched Ptolemaic? – The pivotal epistemological approaches to broaching the subject and solving the renowned problem are commonly laid out by the following significantly diverse options:
(I) naïve inductivist version; (II) falsificationist version of Karl Popper; (III) sober conventionalist version known for the most part thanks to the writings of Pierre Duhem; (IV) distinctive social-psychological version of Thomas Kuhn; (V) sophisticated falsificationist version of Imre Lakatos and Elie Zahar.

Nevertheless, the multifarious explanations for the ultimate reasons for Copernicus's triumph over Ptolemy, though matter-of-course, deft and alluring, seem rather dubious in the light of the following plain counter-arguments.

i. *Inductivist* account famously turns out to be peculiarly fragile because the theories from both competing research projects – that of Copernicus and Ptolemy – *equally* deviated from the available observational data [1]. For instance, still the Buridanists, in their bitter controversy with Aristotle, stressed that, on the solid ground of observations, it is impossible to assert definitely whether the Earth or the stellar sphere moves (pp.521-537) [2,3]. In particular, if the Earth were to have diurnal motion, it would not cause a continuous wind blowing from the east , since the Earth moves also with the water and air.
ii. *Falsificationist* refined explanations of the substantial causes of Ptolemy's defeat are commonly reduced to the following alluring options.

a. According to the best-known one, Ptolemy's deft theory was illicitly irrefutable and therefore unscientific while Copernicus's superlative theory was just the opposite. Ptolemy's notorious heuristic was blatantly *ad hoc*. Any odd celestial fact could be deviously accounted for in retrospect by immense multiplying the inextricable paraphernalia of heterogeneous epicycles, epicyclets, deferents, equants, and so forth.
Nevertheless, the 'unrestricted proliferation' of diverse epicycles in Ptolemaic whimsical astronomy is a wonted 'historical myth' (chapters 11-13) [4]. In actual research practice, to compensate for the flagrant equant triumphal stave off, Copernicus was forced to insert a *new* species of no less stale epicycles. Eventually, Ptolemaic 'antediluvian' program transpired to contain fewer epicycles than the bold 'revolutionary' Copernican one.
b. According to the second, more sophisticated version [5], both competing theories were equally lame for a sufficiently long time. However, eventually, the smashing blow of the 'critical experiment' masterfully refuted Ptolemy and magnanimously buttressed Copernicus. Though when did this staggering miracle happen? Regrettably historians of science do not come to a consensus. Maybe, all appearance, in 1616, when lucky devil Galileo had miraculously detected the cycles of Venus?.

Unfortunately, the common claim that Galileo had successfully predicted the unexpected phases of Venus again constitutes a sought-after historical lapse [6]. One can rightfully sympathize with Galileo's caustic critics among the stout Aristotelians refusing to take seriously the astonishing observational data lavishly produced by a newfangled telescope with rather dubious work principles [7-9].

iii. According to the sober *conventionalist* approach, one cannot dare to make the final choice between the competing theories only grounded on sheer empirical considerations [10]. Eventually, one theory surely fits better than the other because it is more 'simple', 'beautiful', 'coherent', 'economic', etc.

Yet I. Lakatos and E. Zahar fairly retorted that the alluring 'myth of simplicity' was time and oft dispelled by the painstaking labor of the historians of science in variety of case-studies (p.362) [11]. For example, such theory-choice situations are common for the history of science, when one of the competing theories is simpler than the other. However, this 'other' is in better agreement with the available experimental data at the same time or better reconciles with other respectable theories [1]. Which theory should be definitely chosen? The same is true for other subtle conventionalist criteria that in common research practice commonly function as mere values and not as definite and effective criteria.

iv. Due to *T.S. Kuhn's* assiduous accounts (p.367; p.177) [7,12,13], Ptolemy's sophisticated astronomy was going in 1543 through a depressive state of 'paradigm-crisis' that constitutes, according to his thought-provoking epistemological doctrine [13], the indispensable prelude to any scientific revolution.

Nevertheless, as Lakatos & Zahar [11] reasonably parried, how many scholars had perceived this horrific 'community crisis' that, withal, lasted for more than a thousand years? It is no coincidence that one of the mature historians of science expressed the opinion that in the Copernican paradigmatic case Kuhn oddly envisions ' a scandal where there was none' [14]. The scientific community of the time was jolly small and scattered in various convents, universities, cities and countries, not to forget the difficulties of necessary communications. And if Kuhn's mellow analysis of the fine 'structure of scientific revolutions' is ultimately applicable to the Copernican important case, why so *few* scholars had deliberately buttressed Copernicus before Kepler, Galileo, Descartes and Newton?.

Moreover, according to Thomas Kuhn and his partisans, a scientific revolution – in a pivotal analogy to formidable political revolutions, such as the 1793 French revolution or the 1917 Russian one – is taken as resolutely, relentlessly, and completely displacing the '*Ancien* Régime' research traditions in science [15]. Through the course of such rampant breakthroughs, new mature theories are so *radicalized* that they become *"incommensurable*" with their mature predecessors. It leads to the standpoint that the great scholars like Newton, Maxwell, Darwin, Bohr or Einstein , being the instigators of revolutionary breakthroughs, should be pictured as convinced adversaries of the 'creative dialogue' between the 'old' and the 'new' research traditions.

However, such a tenet (together with Kuhn's startling 'imagery of warfare' used to depict the reception of Copernicanism) seems rather unilateral since it apparently exaggerates the revolutionary facet of a profound scientific change.

Likewise, Copernicus aptly *modifies* rather than categorically *rejects* Aristotle's stout principles; moreover, in many important respects he can be regarded just as a discreet reformer of Ptolemaic astronomy rather than an ardent revolutionary intent on its overthrow. For instance, Copernicus modifies famous Aristotelian principle that an object can take part only in a single natural motion. He argues that a more complex principle is required to accommodate falling bodies on a rotating Earth [16].
v. The shrewd SRP (scientific research programs) methodology [17] maintains that the pivotal problem of philosophy/methodology of science is to proffer a *normative*

appraisal of scientific theories. An objective *appraisal* of a scientific change is an especially normative problem and whereupon belongs to analytical philosophy. However, a reliable *explanation* of the scientific change – of the elicited true reasons of acceptance and rejection of the theories involved – is predominantly a psychological/sociological conundrum.

Surely, the Copernican distinctive program showed itself as a 'theoretically progressive' one. This hallmark had been gladly anticipated by one of his disciples:

> "Aristotle says: 'That which causes derivative truths to be true is most true' (Metaphysics 993b 27-27). Accordingly, my teacher decided that he must assume such hypotheses as would contain causes capable of confirming the truth of the observations of previous centuries, and such as would themselves cause, we may hope, all future astronomical predictions of the phenomena to be found true" (Rheticus, Narratio prima, pp.142-143, quoted from [3], p.39).

The Copernican fine program had theoretically accounted for many important 'novel facts' not observed before. For instance, it had successfully yet qualitatively anticipated star parallax. Though *actually*, the Copernican program started to make the conclusive empirical progress only with Sir Isaac Newton (p.374) [11].

The 'Ptolemy-Copernicus' inextricable transition had been masterly reconstructed within the startling methodology of Imre Lakatos. In my view, one should not nitpick to petty lapses that are common for any pioneering treatise of this kind. The crucial point is a *matter of principle*. One should never forget that the ultimate aim of the SRP methodology is to set an *objective and incontrovertible appraisal* of scientific change yet not the sober *explanation* of its *actual* reasons (p.381) [11].

Within the shadowy frame of the Lakatos-Zahar sublime approach, it is quite easy to admit that all the subtle content of the Copernican bold program could be contrived even by Aristarchus of Samos. But why it did not happen? And why did these great designs not have any significant impact on Ptolemy and his numerous followers?.

The near-by goal of the present paper is to strengthen efforts in landing the sky-high Lakatos-Zahar approach by taking a further step on the thorny path of reasonable explanation for the *true* reasons for the incipience and triumph of the Copernican breakthrough program. In my humble opinion, the renowned accounts (I) - (V) unfortunately miss an important (and probably crucial) point of the Ptolemy-Copernicus inextricable transition. Viz., *Copernican and Ptolemaic alternative programs were doggedly realizing the radically different from each other ways of astronomy and physics reconciliation.*

Let us recall how Paul Feyerabend, who devoted more than a dozen years to the study of the Copernican revolution, many a time and oft grievously concluded that *not one reason and not one method, but different reasons, assessed from diverse positions, is what had made up the intricate Copernican revolution*. These reasons and positions were relentlessly intertwined; however, this interweaving was substantially random, so one should *not* try to explain the *whole* medley only by the blunt influence of simplified methodological rules.

Hence the ultimate aim of the present account is to proffer merely a *more refined* (but not the ultimate!) answer to the stale question "*Why did the Copernican bold program eventually supersede the entrenched Ptolemaic one?*" To propose a more apt explanation one has to provide a substantial 'theoretically progressive problem shift' relative to other rival reconstructions and to display that the Copernican Revolution, alas, is a more inextricable phenomenon than seems from the spellbinding conceptions of scientific revolutions [1,18].

In good sooth, the previous accounts have intensely oscillated between *two alternative extremes*. On the one hand, in the common blunt vein, the apparent differences between research traditions were hastily taken to be insignificant and fruitful communication (and even deep interpenetration) unproblematic. On the other hand, in the new-fangled, post-Kuhnian distinctive disquisitions, important differences between the research traditions are commonly exhibited to be so radical that their actual communication is regarded almost impossible.

The present humble account stems discreetly from a more common and ordinary *intermediate picture*. Respectively, I readily admit that the substantial differences between the research traditions existed at the various levels, ranging from entrenched ontological commitments and up to inextricable epistemological beliefs similar to Ptolemy's notorious instrumentalism. Nevertheless, these often antagonistic traditions were able to *communicate* fruitfully in the creative acts of such ingenious men of science as Ptolemy, the Moslem astronomers, Copernicus, Galileo, Descartes, Kepler, and Newton. The research traditions had vigorously communicated by the subtle ways that permitted comparisons, adaptations, and even fruitful *cross-fertilizations*.

The intermediate humble approach originates from the principled criticism of the 'one-sidedness' of Kuhnian and Lakatosian the most advanced epistemological models: they both as a matter of fact (yet due to the different philosophical reasons) lack the subtle mechanisms of the paradigms' (or SRP's) intense *interactions* [19,20]. To meet the critical arguments, a lucid 'mellow theory-change' epistemic model had to be advanced grounded on the *'communicative rationality*' considerations [21].

Respectively, the profound origins of scientific revolutions are fathomed not so much in the stale discrepancies of entrenched mellow theories with the 'hard facts'(common to all SRPs or 'paradigms'), as in the stiff collisions of 'old' pivotal research traditions with each other. The latter transpire in relentless contradictions that *can* be most effectively (yet not always!) excluded in a more general ('global') theory. Whereupon, the leading parts in mature theory change are played by the intense *dialogues* of the proponents of the old paradigms' that lead to mutual accommodation and even profound interpenetration of the participants' s views.

It was sedulously displayed that the global theory piecemeal contrivance is being incessantly dominated by hard internal tensions between the 'old' well-established SRPs. Viz., the inevitable encounter of the stout programs, their deep interpenetration, and strong twisting provide the erection of a vast hybrid realm at first with a haphazard throng of crossbred theoretical models. Gradually, on consecutive soothing and eliminating the contradictions between the hybrid models, the crossbred solid system is sedulously set up.

Up to a point, the abovementioned epistemic model can be taken as proffering a reliable *mechanism* of practically simultaneous drastic transformation of the 'old' paradigms. The hallmark of the sophisticated mechanism is the contrivance of the crossbred systems constructed from the basic theoretical objects of 'old' mellow theories. The crossbred systems constitute cohesive channels through which the accommodation of the 'old' traditions encountered relentlessly commences. The accommodation gradually leads to the installation of a throng of *crossbred* theoretical schemes. The hotchpotch will be ultimately processed and subsequently generalized to set up a conspicuous *structure* of a novel global theory. Incidentally, a mellow scientific theory may be empirically successful if it constitutes a kind of a 'draft' that can be furnished and explicated by the results of new bold experiments that increase its empirical content significantly. The global theory should become such an effective vehicle for the production of new testable statements. For this purpose, it should reconcile and intertwine the 'old' classical research traditions in such a refined way that they once and for all cease to contradict each other, so that the newly-constructed synthesis can successfully explain and anticipate novel experimental evidence.

Hence the crux of the present account is to provide further reliable historical backing to the lucid epistemological theory-change model [21]. We contend that *profound breakthroughs in science were first and foremost not due to ingenious contrivances of brave novel paradigms or bold invention of startling new ideas 'ex nihilo' (*the gist of the Copernican program was known even to Aristarchus!*). On the contrary, the breakthroughs were caused by the harrowing humble processes of piecemeal accommodation, gradual interpenetration, and discreet intertwinement of the 'old' pivotal research traditions preceding such radical breaks.*

For instance, in creating his epoch-making 1905 masterpieces (the daring theory of light quanta plus the whimsical special relativity) Einstein was inspired by a stout belief in the necessity for *unity* in science [22] - the faith he carried through his whole life. Whereupon, sagacious identifying and dexterous resolving the relentless paradoxes revealing the inexorable contradictions between the entrenched research traditions turn out an indispensable part of the Scientific Method as such. A case of diverse programs' encounter leads to a wonted situation when a domain of *hybrid* models occurs formed by plain conjunctions of the models of different research programs (Ptolemy, Ticho Brahe, Lorentz, Abraham, Klein). However, commonly the hybrid models transpire to be self-contradictory; and when this is properly realized (Copernicus, Maxwell, Einstein, Dirac), the crossbreeds are deftly constructed from the basic objects of *all* the cross-theories. *The contrivance of new mellow theory commences owing to the crossbred domain's gradual growth.*

The present study strives to exhibit that the Copernican breakthrough turns out a result of elucidation and (partial) resolution of the profound dualism, of the deep abyss between Ptolemy's deft mathematical astronomy and Aristotelian descriptive qualitative physics. Therefore, it is not accidental that the dazzling writings of Copernicus, Galileo, Kepler, Descartes, Newton, and their disciples were all the indispensable levels of mathematics descendance from Divine heavens to sinful Earth and the reverse conquest of the Sky by Earthly mature physics.

The Rise and Extinction of Ptolemaic Sophisticated Research Program

Lakatos and Zahar carefully described how Ptolemy and Copernicus had coherently advanced the diverse *research programs*. In particular, both rival designs branched off from the same Pythagorean – Platonic 'protoprogram'. Its dominating principle constituted that, since heavenly bodies are immaculately perfect, *all the astronomical appearances should be deftly 'saved' by the artful combinations of as few uniform circular motions as possible.* The resolute principle was installed as the firm cornerstone of the powerful *heuristic* of both programs; wherein the '*heuristic*' was principal, while the '*hard core*' only subordinate. The hard core of Ptolemy's program was made up from the geocentric solid hypothesis in conjunction with the Aristotelian natural philosophy, with its paraphernalia of natural and violent motions and the impenetrable and stout demarcation line between the shabby terrestrial (sublunary) and sublime celestial realms. Though Aristotelian physics was an empirical science *par excellence* that imbibed common experience much more naturally and fully than the rigid, strict, elevated, and abstract science of Galileo, Descartes, and Newton. Everyone knows too well that hard bodies quite naturally and often unexpectedly fall down, while fire cheerfully and lightly soars up.

Regrettably, inertial motion is not an experimental fact at all: common experience apparently contradicts it. Furthermore, everyone knows quite well that in Nature 'there is no void existing separately' [23]. The Sun and the Moon relentlessly rise and set, while the thrown bodies do not conserve their rectilinear motion.

Due to common human experience, masterly fixed by the flexible notions of Aristotelian natural philosophy, the distinctive '*lebenswelt*' (Husserl) in which one lives and acts mercifully is not a mathematical one nor can it be successfully 'mathematized'. This volatile kind of reality is the wonted realm of uncertainty, unsteadiness, inaccuracy, in an appropriate manner described by the vague terms 'almost', 'a kind of', 'more or less', etc. Whereupon the Greek sober thought could hardly concede that exactness can successfully survive in the wry world around us and that the ponderous matter of stale sublunary world with its inextricable medley of four basic elements (earth, water, air, and fire) can adequately 'represent mathematical entities' [24] similar to the ideal objects of Euclidean geometry. For obvious reasons, thanks to Stagiritul, 'the [immaculate] mathematical exactness should be demanded only for the [peculiar] objects lacking matter' [25].

On the contrary, the heavens are erected from an entirely different, odd, and immutable substance, the mysterious 'aether' (or the renowned 'fifth element'). Heavenly bodies are inextricable components of queer ether spherical shells that fit tightly around each other in a strict order that begins with the Moon, and extends to the sphere of distant fixed stars. Each heavenly shell is characterized by its specific rotation, which accounts for the peculiar motion of the heavenly body contained in it. At the same time, outside the utmost sphere of the fixed stars, the powerful *prime mover* is solidly situated. This one diligently imports motion from the outside sphere inward. Eventually, the natural motions of all the heavenly bodies are unforced, perfectly circular, and neither speeding nor slowing down.

Clearly, the perfect motions of the stars take their place in accordance with strict geometrical laws. Thus, thanks to Stagiritul's discreet doctrine, *mathematical astronomy is apparently possible while mathematical physics is certainly not.* Whereupon the masterful Greek astronomers not only applied sophisticated mathematics but with amazing patience and magnificent skill observed the skies. Though they had not even dared to mathematize inextricable terrestrial motions.

Claudius Ptolemy (87-150) famously reached the acme of ancient science not least because he was a Hellenistic astronomer, astrologer, mathematician, geographer, and even poet. His chef-d-oeuvre – the celebrated '*Almagest*' [26] - stiffly dominated the European thought for more than fourteen centuries. The social-cultural context of the Ptolemaic intricate research program was determined by the eloquent fact that the author of '*Almagest*' was a successful resident of Alexandria, the splendid capital of Hellenized Egypt. Hellenistic conspicuous civilization was a distinctive blend of stunning Greek culture with the most ancient civilizations of Egypt and Babylon and a substantial departure from the earlier arrogant Greek attitudes towards the dubious 'barbarian' cultures. The extent to which genuinely hybrid Greco-Asian cultures emerged is rather contentious, but it is indisputable that the relentless encounter of the three cultures captured even the elevated domain of theoretical astronomy [27].

Unlike the 'antediluvian' Babylonians and Egyptians, who sedulously studied the heavens merely to keep track of their shabby seasons, the sophisticated Greeks considered astronomy from a sublime theoretical standpoint: they strived to comprehend the basic nature and makeup of the whole Universe. Claudius Ptolemy, as a typical Hellenist, *craved* (not without success) *for balancing himself discreetly between the three distinctive cultures* in all the basic domains of research activity but first and foremost in ontology and epistemology. In natural philosophy, he deftly oscillated between sophisticated mathematical models and blunt, empirically-grounded qualitative physics, while in epistemology – between the condo realism and quasi-Duhemian crude instrumentalism.

On the one hand, he was certainly aware yet decidedly skeptical of Aristarchus' quirk heliocentric hypothesis due to quite rational arguments grounded on the stout and well-grounded empirically principles *of Aristotelian physics*

On the other hand, since Aristotle turned to be the only philosopher to whom Ptolemy referred explicitly, the notorious author of the '*Almagest*' was hastily accused of slavish adherence to the blunt principles of Aristotelian natural philosophy. However, the perfunctory dictum is an obvious oversimplification. One should never forget that, according to Aristotle (De Caelo, ch. V), the rest is nobler than the motion and the Sky is much more perfect and nobler than the Earth. Further, the '*Almagest*' immaculate model of the solar system did *substantially* deviate from orthodox Aristotelianism at least in the two crucial respects.

i. Aristotle persistently maintained that heavenly bodies should move around the Earth in *single* uniform circles. Yet, in the Ptolemaic inextricable models, the odd motion of the planets is an intricate *combination* of *two* circular motions; moreover, at least one of them is *non-uniform*.
ii. Aristotle also asserted – alas, again from the elevated and abstract ontological grounds – that the Earth should be located at the *exact* center of the Universe. But in the Ptolemaic feasible distinctive system, the Earth is decidedly *displaced* from the center of the Universe. Moreover, in Ptolemy's inextricable cosmology there is no unique center of the Universe; the central orbit of the Sun and the planetary local deferents all have slightly *different* geometric centers, none of which coincides with the Earth. Incidentally, in the '*Almagest*' Ptolemy had frequently shown off that the non-orthodox (concerning the Aristotelian natural philosophy) aspects of his subtle models all were directly dictated by stubborn facts of observations.

Whereupon, Ptolemy's most controversial contrivance was the famous '*equant*': a planet revolves around the Sun at a non-uniform rate. Nevertheless, it can facilely be exhibited that the non-uniform rotation of the radius-vector connecting the planet to the Sun implies a uniform rotation of the radius-vector connecting the planet to the so-called 'equant'. The latter is the peculiar point directly opposite the Sun relative to the geometric center of the orbit [28].

It is merely from the modern historically one-sided standpoint that Ptolemy's notorious equant appears a precursor of the textbook Kepler ellipses. Yet for such caustic yet objective critics as the Islamic /Arabic astronomers and subsequently Copernicus (since the works of the Maragha school might be known to the author of '*De Revolutionibus*') [29] and his associates, the compelled insertion of the equant was a typical Lakatosian 'ad hoc_3 hypothesis'. It obviously contradicted the respectable spirit of the Aristotle - Ptolemy program – the pivotal tenet of uniformity of motion in respect to the center of the Universe. Let us recall that, according to Lakatos & Zahar's alluring stance, the 'ad-hocness' should be taken, within the flexible frames of the SRP methodology, not as a narrow property of an isolated hypothesis but as a multilateral *relation* between two consecutive theories. Incidentally, it had been Ibn al-Haytham (965-1040) who initiated the first critique of Ptolemy's physics for the violation of the basic principle of uniform circular motion.

It is no coincidence that after Eudoxus' primitive model (made up of the system of concentric rotating spheres) was decidedly abandoned, any actual progress in the realization of geostatic program ran counter to the powerful heuristic of Platonic protoprogram [30]. The eccentric stiffly displaced the Earth from the center of the circle; the Apollonian and Hipparchan deft epicycles provided that the actual path of the planets about the Earth was not circular; and, eventually, the Ptolemaic notorious equants entailed that even the motion of the epicycle's empty center was not simultaneously uniform and circular. The perfidious insertion of the equant was the heaviest blow upon the stale heuristic of the Platonic protoprogram: it was almost equal to its full wreck. Thus, within the Ptolemaic intricate program *mathematical exactness that insisted on the insertion of noncircular orbits and the centers of rotation not coinciding with the earth center began to diverge increasingly and irreversibly from the stout and empirically sound principles of Aristotelian physics.* Hence in the long run one can take *Ptolemaic inextricable cosmology as a substantially dual fundamental theoretical scheme waywardly mixing the principles of 'Platonic immaculate mathematics' with those of 'Aristotelian sober physics'.*

To retain the indubitable advances of the Aristotelian dazzling doctrine and the conspicuous achievements of the mathematical astronomy, Ptolemy in the '*Planetary Hypotheses*' had to advance further and at the same time soften the blatant rupture between the mundane and celestial phenomena. Since the '*Almagest*' confined itself to subtle mathematical models, Ptolemy had to proffer preliminary physical models for the same constructions, yet in vain. One of the apparent examples of his numerous failures is represented by the attempt to construct the ingenious 'tambourine-like', 'sawn-off' mechanism able to transform motion from outer spheres of his cosmos to inner ones [31].

Alas, *after the 'Planetary Hypotheses' the deep abyss between the physical and mathematical facets became even deeper* [32]. So, Ptolemy's inextricable cosmology could not help but be exposed to severe repeated attacks during the European Middle Ages [33]. It obviously confronted the strict principles of monotheism not admitting the stiff demarcation line between the celestial and mundane worlds; *all seemingly different worlds nevertheless should have one and the same Creator.*

The Copernican Abstruse Breakthrough: An Origin and Advancement

It was Alexander Koyré who turned to the asset of the pivotal cross-contradiction necessary to comprehend the subtle intertheoretic context of the Copernican revolution again. Koyré was a thought-provoking French / Russian epistemologist, philosopher and historian of science, renowned for neo-Hegelian standpoint, as well as for his profound influence on Thomas Kuhn's celebrated concept of structure of scientific revolutions.

Not by chance it was an 'internalist' Alexander Koyré who in the middle of the XX-th century became increasingly aware of the '*tremendous gap*' between mathematical astronomy and Aristotelian qualitative physics inherent to ancient cosmology. Respectively, the ultimate motive of contrivance of the heliocentric program consisted not at all in sedulous elimination of the wonted discrepancies between the Ptolemaic 'wry' cosmology and stout observational 'hard facts'. (Forsooth, according to Imre Lakatos, experimental anomalies are inherent in all stages of development of a scientific theory).

Accordingly, in part III of his punctilious monograph , André Goddu contended that Copernicus can rightfully be called a '*principled thinker'* [16], for he broached and resolved first and foremost *principal questions* while ignoring secondary ones. Though he was not a systematic philosophical thinker; philosophy in due course taught at his alma mater - Cracow University – in the last quarter of the 15th century can be characterized as 'eclecticism'. Yet he strived to maintain a clear sense of his principal goal, and to distinguish properly between the questions that he had to address and ones he could leave aside.

Moreover, in my humble view, Copernicus was invigorated predominantly by aesthetic and metaphysical (and, I insist*, essentially theological*) considerations aimed at effectively eliminating the profound rupture between refined mathematical astronomy and Aristotelian blunt qualitative physics. However, it did not prevent him from applying standard Aristotelian 'argumentative technique' to demonstrate the greater probability of his assumptions over those of his geocentric opponents.

For instance, "all these arguments make it more likely [more probabilior] that the earth moves than that it is at rest. This is especially true of the daily rotation, as particularly appropriate to the earth" (*De Revolutionibus*, book I, ch.8, p.17).

But certainly it was his Christian Weltanschauung that made the rupture especially bigoted. Namely this Weltanschauung moulded the staff provided by the three basic sources: the cosmological views of Pythagoreans, Aristotle's fine cosmology [34] and (last but not least!) the inexorable its critique provided by the Buridan's school embracing John Buridan, Nicole Oresme, and Albert of Saxony.

"Copernicus developed a critical approach to natural philosophy but one that enabled him to modify Aristotelianism while remaining a participant in a broadly conceived Aristotelean tradition [...] He developed his own version of Aristotelianism, very much influenced by Renaissance Platonism, Neoplatonism, Stoicism [...] It was also a version of Aristotelianism that was anti-Averroistic and more flexible in its approach to mathematics" (pp. xxii, xxiv) [16].

In that respect Copernicus can be regarded as a scholar which belonged to a respectable ancient tradition of reconciling Ptolemy's mathematical models with the concentric cosmology of Aristotle [34] though in favorable socio-cultural context of Christian Weltanschauung enforcement.

Whereupon, the celebrated introduction to his opus Magnus - '*De revolutionibus orbium coelestium'* (1543) - was by right dedicated to 'his holiness Pope Paul III'. The honorable canon at Frombork Cathedral, Doctor of Theology, whose maternal uncle (and powerful promoter) was Lucas Watzenrode, the honorable bishop of Warnia, openly admits that he had been relentlessly "impelled to consider a different [from Ptolemy] system of deducing the motions of the universe's spheres for no other reason than the realization that *astronomers do not agree among themselves in their investigations of this subject".*

Furthermore, "those who devised the eccentrics seem thereby in large measure to have solved the problem of the apparent motions with appropriate calculations. But meanwhile, they introduced a good many ideas which *contradict the first principles of uniform motion.* Nor could they elicit or deduce from the eccentrics the *principal consideration*, that is, the structure of the universe and the true symmetry of its parts".

On the contrary, "we discover a marvelous symmetry of the universe, and an established harmonious linkage between the motion of the spheres and their size, such as can be found in no other way" (p.22) [35].

Note that Copernicus' Christian Weltanschauung determined his especially discreet attitude towards the sober arguments of the pagan Aristotle. Therefore, he is not limited to referring to the authority of Aristotle to justify the unacceptability of uneven movement. Instead, he works out a stout metaphysical argument of *his own*: variable speed can only be caused by variable force. But God, as the Primordial Cause of all the movements, is Always Constant. Consequently, "I began to be annoyed that the movements of the *world machine*, *created for our sake by the best and most* *systematic Artisan of all,* were not understood with greater certainty by the philosophers, who otherwise examined so precisely the most insignificant trifles of this world" [35].

Not by chance one of the pivotal chapters in Rheticus' '*Narration prima*' wore the meaningful headline "The Principal Reason Why We Must Abandon the Hypotheses of the Ancient Astronomers".

Secondly, while for Stagiritus gravitation constitutes the tendency of heavier bodies to long to the center of Universe, for Copernicus it is merely the tendency of heavier bodies to get to the centers of any spherical masses of matter.

The true origin of the inextricable paradoxes, due to Copernicus's sublime standpoint, consists in the non-ideal odd movement of the planets. However, according to the Aristotle - Ptolemy sober doctrine, resolutely buttressed by the Christian faith, the planets necessarily belong to ideal spheres and *should* be engaged in uniform motions along with the perfect circles or along with their artful combinations. Following not so much Stagiritul as his own theological arguments, Copernicus was convinced that the supposed perfection of the heavens requires celestial bodies to execute *uniform* circular motion only. Whereupon he was spurred to decisively reject first and foremost the dubious equant model.

The drastic rejection of the equant was connected with that Copernicus not only employed a great deal of observational data from Arabic astronomical sources, but also many subtle theoretical components as well. The latter first and foremost included the famous 'Tusi couple', used by Copernicus to exchange Ptolemy's geometrical construction of the equant model. Thus he willingly agreed with the sharp criticism of Ptolemy's model by Ibn al-Haytham, Ibn Rushd, Nur as-Din al-Betrugi and other Moslem astronomers [36].

In '*De Revolutionibus*' Copernicus realizes the task targeted already in the '*Commentariolus*'.The motivation for creating the heliocentric system consisted in the assertion, that if the planetary orbs are ordered around a single center according to a strict principle, then the Earth cannot be that center. Eventually, ultimately inspired by the '*best intentions to layout the Divine Order of the Heavens*' Copernicus craves to return to Aristarchus's queer proposal to place the center of the Universe on the Sun. But namely this generated the profound paradoxes within the Aristotelian physics inextricably connected with the stale notions of 'natural' and 'violent' movements. As a result, Copernicus had gradually constructed merely a sophisticated *crossbred* theory that incepted to pave the way to divine immaculate mathematics and mundane qualitative physics profound *interpenetration*. As modern French historian punctuated, "Copernicus in an insinuating manner and probably unconsciously had inserted into the stout Aristotelian fortress two innocent premises through which Kepler, Galileo, and Descartes vigorously blew it up" (p.430) [9].

In the stupendous Introduction to '*De Revolutionibus'*, Copernicus passionately appeals to a refined, influential *coterie* of respectable clergy including Pope Paul III (to whom the Opus Magnus was dedicated), Pope Clement VII (who not only adopted the '*De Revolutionibus*' but even insisted on its publication), Nicholas Schonberg, cardinal of Capua, Lucas Watzenrode, bishop of Warmia, Tiedemann Giese, bishop of Chelmno, et al. Throughout the whole book the author of '*De Revolutionibus*' consistently condemns the author of '*Almagest*' for inherent *paganism*. In short, his scathing criticism of Ptolemy's stale system is grounded on the weighty argument, according to which the Pagan's inextricable medley system (deftly embracing tens and hundreds of epicycles, abstruse epicyclets, and quirk equants) is nevertheless lacking strict, monotheistic Order, pre-established by the Lord, 'the best and *most systematic* Artisan'. Incidentally, sometimes the instrumentalism of the notorious '*Almagest*' went so far that one and the same movement of the same planet was described by two *substantially different* mathematical models (see, for instance, Ptolemy's queer math models of Mars's odd motion). Whereupon, the multifarious components of the Ptolemaic motley system vividly epitomize the various plans of different pagan 'artisans', of warring Hellenistic gods, densely inhabiting both dazzling Olympus and the dismal tombs of the pharaohs.

Incidentally, "their [the Ptolemy partisans'] experience was just like someone taking from various places hands, feet, a head, and other pieces, very well depicted, it may be, but not for the representation of a single person; since these fragments would not belong to one another at all, a monster rather than a man would be put together from them" [35].

Similarly, Copernicus – probably paved the broad way for Galileo's and Newton's mighty mathematical physics. If the strict and robust demarcation line between divine and mundane worlds is lacking since the Earth is just an ordinary planet of the Solar system, the mathematical subtle notions and elevated principles should apply both to its rotations around its axis and the Sun, as well as to all the bodies moving along its surface. It is no coincidence that in subsequent Galileo's works, the graphical Aristotelian 'natural' movements had to be decidedly transformed into abstract and sublime 'inertial' ones.

For obvious reasons, the quirk heliocentric picture of the world seemed incredible to most sane people of that time. For instance, if the Earth actually revolves around its axis and even around the Sun, how are celestial bodies held on the planet's surface? - Copernicus himself cleverly, though unfoundedly tried to get out of the stumbling block using a typical ad hoc_2 hypothesis: each celestial body can have its own, local, (of incomprehensible origin) gravity. Therefore, all heavy objects should gravitate towards the local centre.

Galileo commenced with descending mathematics from the Skies to the Earth, being inspirited not so much by '*De Revolutionibus*' as by Plato's '*Timaeus*' (and the corresponding renowned yet controversial discoveries provided by the new-fangled telescope). Indeed, if the Earth is just one of the ordinary planets, then the laws of mathematics, previously applied to describe the motion of everything that happens in the supra-lunar world, now apply to its movement as a whole, and to what happens on its surface as well. As a true Copernican, Salviati points out in the '*Dialogues*' in his dispute with the stubborn Aristotelian Simplicio, "and [as] to the Earth, we try to ennoble it and make it more *perfect,* striving to *liken it to celestial bodies* and, in a sense, to place it in heaven, from where your philosophers expelled it" [38].

In path-breaking '*The Assayer*' [37] Galileo famously asserts that "philosophy is written in that great book which ever lies before our eyes — I mean the universe — but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written**.** This book is written in the mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible to comprehend a single word of it" (p.75, [39]; see also Drake's distinctive translation pp.237-238, [40]).

And to calm down the reader by whom the magnificent Book was written, in his renowned introduction to the '*Dialogue Concerning the Two Chief Systems – Ptolemaic and Copernican*' Galileo points out that "he who looks the higher is the more highly distinguished, and turning over the Great Book of Nature (which is the proper object of philosophy) is the way to elevate one's gaze. And though whatever we read in that book is the creation of the omnipotent Craftsman, and is accordingly excellently proportioned, nevertheless that part is most suitable and most worthy which makes His work and His craftsmanship most evident to our view" [38]. Eventually (for faithful Galileo) immaculate and dazzling "*mathematics is the language with which God has written the Universe* "(p.2) [41].

Koyré insisted that Galileo's alluring interpretation of Christian theology was inspired by Plato's '*Timaeus*'; especially by the whimsical myth of the creation of the Universe. The key figure of the renowned dialogue – the almighty Demiurge, a divine Craftsman – punctiliously constructs the stout mathematical order out of the preexistent chaos to put up the Universe (the 'cosmos'). For that grand purpose he punctiliously cuts out small triangles to erect four regular solids; and then he artfully applies them to construct real bodies, plants, and even animals out of them [42]. Moreover, it was in '*Timaeus*' where the notion of a divine Craftsman was enriched by the notion of pre-established harmony devised by him.

Thus, Nature transpires to be empty and ordered: in the process of its creation God discreetly put strict mathematical necessity in it. Hence mathematical knowledge is not merely true but is substantially *sacred* even more than the Holy Bible. While there are plenty of interpretations of the Bible, the mathematical truths are unique and should be relentlessly kept out of discussions.

At the sake of mathematization, Galileo had radically transformed the subtle methodology of natural science: he had relentlessly elevated mathematization, as well as (real and thought) experimenting up to the highest ranks of leading scientific methods [43]. Eventually, it made it possible for the Florentine to contrive the paramount 'principle of inertia' and even to come close to the second law of Newton's dynamics [44].

Yet the very opportunity of implementation of mathematical methods in natural science turned out to be grounded on the wayward procedure of *idealization*. Correspondingly, the modern exact science commenced with taking all natural phenomena as more or less adequate approximations of some Platonic 'ideal essences'. The latter lack, contra Aristotle, profound existence *within* the natural phenomena but exist *alongside* them as the 'certain limits of infinitely small sensory becoming'; hence they can be freely contrived by the ingenious human mind. And they are the stiff relations between the ideal 'quasi-essences' that are depicted by the exact Laws of Nature. At the same time, the relations between *real* objects (e.g. rods and clocks / tables and chairs) are exhibited merely by the *approximations* to the strict laws. Just as Galileo had succinctly put it, 'the search for essences, in my judgment, is a vain and hopeless kind of pursuit".

Consimilar platonic (and neo-platonic) motives that found their distinctive expression in the alluring tenet of '*delightful accordance between the [base] Cosmos and the Holy Trinity*' induced Kepler to the strenuous search for the stout mathematical laws stiffly governing the planet motions. Properly educated in strict Lutheran faith, Kepler had passionately devoted himself to delve into the sacred '*Book of Nature'*.

"We astronomers are priests of the highest God concerning the book of Nature" [45].

But the innovative views of Kepler and Copernicus differed significantly. In Copernicus's plain theory, the planetary motions were impeccably circular; they demanded no causes and took part due to the inertia of the bodies. Hence the Sun was not the actual center of force. Only Kepler's distinctive reflections on the true source of planetary motion revealed the leading role of the Sun and buttressed him in describing the subtle mechanics of planetary motion.

On the firm and stout grounds of Trinitarian thoroughgoing doctrine, Kepler decidedly took the Sun as the geometric and dynamical center of the cosmos. The coequality of Father, Son, and Spirit implied the continuity of the Center, Periphery, and space of the Cosmos. Herewith the Sun epitomized God the Father, the Stars respectively referred to God the Son, while the wonted planets humbly incarnated the Holy Ghost. Furthermore, Kepler plied to discover an immaculate universal law that would meticulously picture the motion of both the Earth and the planets. His relentless quest was invigorated by the alluring analogy between the base Cosmos and the Holy Trinity.

Whereupon Kepler took the second drastic step towards the deep interpenetration of mathematical astronomy and qualitative physics and elicited the laws roughly breaking the Aristotelian stiff principle of uniform rotation of divine bodies. His three laws were famously the first scientific laws taking mathematical form. The Skies relentlessly commenced ruining the qualitative physics of Aristotle. Fruitful reconciliation of the divine and sublunary realms masterly moved Aristotelian physics aside [46].

Incidentally, the epistemological essence of the Copernican-Galilean scientific revolution (unification of the celestial and terrestrial physics) found its startling expression in the title of Kepler's masterpiece: '*Astronomia Nova seu Physica Coelestis'* (The New Astronomy of Physics of the Heavens).

Kepler was not alone in rejecting Aristotle's natural philosophy from the vantage standpoint of Christian theology. In 1644 Rene Descartes had published '*Principia Philosophiae*' where he proposed a theory of motion *directly* grounded on heliocentric model. However, while Copernicus and Galileo were unable to properly explain the physical processes that resulted in the motion of planets around the Sun, Descartes had successfully hit the target relying on the arguments on 'God's immutability'. In his thoughtful doctrine, motion occurred thanks to divine intervention [47]. Thereafter, conservation of motion existed thanks to immutability of God's creation; the immutability of God surely abhors vacuum. Any disturbance causes vortexes to appear, which is the ultimate reason why the Sun and the planets exist, and why the planets obediently rotate around the Sun. Eventually, Descartes' masterpiece epitomized the final elimination of the Aristotelean stale system. According to modern historians, it was Cartesianism which turned the queer heliocentric surmise into a dominant paradigm [9].

Sir Isaac Newton's main purpose was to elicit the stout laws that firmly dictate the motion of *both* terrestrial and divine bodies. He had pioneered in demonstrating, - thanks to powerful heuristic of Copernicus and Galileo, - that it was the same force that attracted all the bodies to the Earth that compelled the Moon to obediently orbit the Earth.

Sir Isaac masterly amended the 'hard core' of the Copernican celebrated program by efficaciously unifying and generalizing the partial theoretical schemes of Copernicus, Kepler, Hook, Descartes , and Galileo and coming up to the whimsical conjunction of three laws of dynamics with the mysterious gravitation law.

Eventually, in the powerful science of modernity Aristotelian qualitative 'essences' had been ingeniously replaced with immaculate mathematical abstract objects. This is especially clear in Newton's "*Principia.*

"Since the ancients (as we are told by Pappus) made great account of the Science of Mechanics in the investigation of natural things; and the moderns, laying aside substantial forms and occult qualities, have endeavoured to subject the phenomena of nature to the laws of mathematics; I have in this treatise cultivated Mathematics, so far as it regards Philosophy" (p.1) [48].

In Newton's resolute methodology the relentless dictum 'to subject the phenomena of nature to the laws of mathematics' constitutes the most robust one. A man of science should while sedulously examining the intricate phenomena of nature, force his sense data in such a dry and suitable for experimental purposes way as to proffer them for immaculate analytical treatment. Exactly in this way the basic mathematical abstract objects of classical mechanics, beginning with a material point, were gradually contrived [49-53].

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