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The Reducibility of Generalized Syllogisms with the Quantifiers in Square {*not all*} and Square {*most*}

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Abstract

To explore the reducibility of non-trivial generalized syllogisms with the quantifiers in Square {*not all*} and Square {*most*}, this paper first gives the formalization of generalized syllogisms on the basis of set theory, and then proves the validity of the generalized syllogism *EMO-3* by first-order logic and generalized quantifier theory; Finally, with the help of some reductive operations, the other 20 valid generalized syllogisms are deduced from the syllogism *EMO-3*. In other words, there are the reducible relationships between/among the 21 valid syllogisms. The reason for this is that any quantifier in a square can define the other three quantifiers. This research method is applicable to the study of syllogisms with quantifiers in other squares.

Keywords: Generalized syllogism; Square{*not all*}; Square{*most*}; Reducibility

Introduction

Syllogism reasoning characterizes the semantic and reasoning nature of the quantifiers it involves, playing a significant part in natural language and human thinking [1]. Various kinds of syllogisms are frequently used in natural language, such as categorical syllogisms [2], generalized syllogisms [3,4], Aristotelian modal syllogisms [5,6], generalized modal syllogisms [7], and so forth. In generalized quantifier theory, a modern square consists of a quantifier and its three negation quantifiers [8]. Let Q be a generalized quantifier, and $\neg Q$, $Q\neg$, and $\neg Q\neg$ respectively stand for its outer, inner and dual negative quantifier. Square $\{Q\} = \{Q, \neg Q, Q\neg, \neg Q\neg\}$ shows the square composed of the four quantifiers. For example, Square {*not all*} = {*not all, all, some, no*}, and Square {*most*} = {*most, at most half of the, fewer than half of the, at least half of the*}. This paper concentrates on studying the reducibility of the generalized syllogisms which include at least one quantifiers in Square {*not all*} and Square {*most*}.

Preliminaries

Let *g*, *t* and *y* be the lexical variables in a generalized syllogism, and the sets composed of the three variables are *G, T,* and *Y*, respectively. Let *D* be the domain of the lexical variables, $G\cap Y$be the cardinality of the intersection of the set *G* and *Y*, and β, λ, σ and τ be well-formed formulas (noted as wff). '$\vdash\lambda$' means that the wff λ is provable, and '$\beta=_{def}\lambda$' that β can be defined by λ.

The generalized syllogisms discussed in this paper only involves eight quantifiers in Square {*not all*} and Square {*most*}, which correspond to the following eight propositions: 'Not all *gs* are *ys*', 'All *gs* are *ys*', 'Some *gs* are *ys*', 'No *gs* are *ys*', 'Most *gs* are *ys*', 'At most half of the *gs* are *ys*', 'Fewer than half of the *gs* are *ys*', 'At least half of the *gs* are *ys*'. They are abbreviated *as not all(g, y), all(g, y), some(g, y), no(g, y),* *most(g, y), at most half of the(g, y), fewer than half of the(g, y), at least half of the(g, y)*, respectively. And they can be denoted by Proposition *O, A, I, E, M, H, F, S*, respectively.

Example 1:
Major premise: No penguins are flying animals.
Minor premise: Most penguins are animals living in Antarctica.
Conclusion: Not all animals living in Antarctica are flying animals.

Let *t, y,* and *g* be variables representing a penguin, a flying animal and an animal living in the domain, respectively. Thus, this syllogism can be formalized as *no (t, y)* ∧ *most (t, g)* → *not all (g, y)* and abbreviated as *EMO-3*. In fact, there are countless instances of natural language corresponding to the generalized syllogism *EMO-3*.

Formal System of Generalized Syllogisms

The formal system of generalized syllogisms generally includes primitive symbols, basic axioms, formation rules, deductive rules, etc.

Primitive Symbols

Brackets: (,)
operators: ¬, →
quantifiers: not all, most
lexical variables: *g, t, y*

Basic Axioms

A1: If β is a valid formula in first-order logic, then ⊢β.
A2: ⊢*no(t, y)*∧*most(t, g)*→*not all*(*g, y*) (namely, the syllogism *EMO-3*).

Formation Rules

1. If *Q* is a quantifier, *g* and *y* are lexical variables, then *Q(g, y)* is a wff.
2. If β is a wff, so is ¬β.
3. If β and λ are wffs, so is β→λ.
4. Only the formulas generated by the above three rules are wffs.

Deductive Rules

Rule 1 (subsequent weakening): From ⊢(λ∧σ→τ) and ⊢(τ→β) infer ⊢(λ∧σ→β).
Rule 2 (antecedent strengthening): If ⊢(β→λ) and ⊢(λ∧σ→τ), then ⊢(β∧σ→τ).
Rule 3 (antecedent strengthening): If ⊢(β→σ) and ⊢(λ∧σ→τ), then ⊢(λ∧β→τ).
Rule 4 (anti-syllogism): From ⊢(λ∧σ→τ) infer ⊢(¬τ∧λ→¬σ).
Rule 5(anti-syllogism): From ⊢(λ∧σ→τ) infer ⊢(¬τ∧σ→¬λ).

Relevant Definitions

D1 (conjunction): (λ∧σ) =$_{def}$¬(λ→¬σ);
D2 (bi-condition): (λ↔σ) =$_{def}$(λ→σ)∧(σ→λ);
D3 (inner negation): (Q¬)*(g, y)* =$_{def}$*Q(g, D−y)*;
D4 (outer negation): *(¬Q)(g, y)* =$_{def}$It is not that *Q(g, y)*;
D5 (truth value): *all (g, y)* =$_{def}$*G*⊆*Y*;
D6 (truth value): *some (g, y)* =$_{def}$*G*∩*Y*≠∅;
D7 (truth value): *no (g, y)* =$_{def}$*G*∩*Y*=∅;
D8 (truth value): *not all (g, y)*=$_{def}$*G*⊈*Y*;
D9 (truth value): *most (g, y)* is true iff *G*∩*Y*>0.5*G* is true;
D10 (truth value): *at most half of the* (*g, y*) is true iff *G*∩*Y*≤0.5*G*;
D11 (truth value): *fewer than half of the* (*g, y*) is true iff*G*∩*Y*<0.5*G* is true;
D12 (truth value): *at least half of the* (*g, y*) is true iff *G*∩*Y*≥0.5*G* is true.

Relevant Facts

Fact 1 (inner negation):
(1.1) ⊢*all(g, y)*↔*no*¬*(g, y);*
(1.2) ⊢*no(g, y)*↔*all*¬*(g, y);*
(1.3) ⊢*some(g, y)*↔*not all*¬*(g, y);*
(1.4) ⊢*not all(g, y)*↔*some*¬*(g, y);*
(1.5) ⊢*most(g, y)*↔*fewer than half of the*¬*(g, y);*
(1.6) ⊢*fewer than half of the(g, y)*↔*most*¬*(g, y);*
(1.7) ⊢*at least half of the(g, y)*↔*at most half of the*¬*(g, y);*
(1.8) ⊢*at most half of the(g, y)*↔*at least half of the*¬*(g, y).*

Fact 2 (outer negation):
(2.1) ⊢¬*all(g, y)*↔*not all(g, y);*
(2.2) ⊢¬*not all(g, y)*↔*all(g, y);*
(2.3) ⊢¬*no(g, y)*↔*some(g, y);*
(2.4) ⊢¬*some(g, y)*↔*no(g, y);*
(2.5) ⊢¬*most(g, y)*↔*at most half of the(g, y);*
(2.6) ⊢¬*at most half of the(g, y)*↔*most(g, y);*
(2.7) ⊢¬*fewer than half of the(g, y)*↔*at least half of the(g, y);*
(2.8) ⊢¬*at least half of the(g, y)*↔*fewer than half of the(g, y).*

Fact 3 (subordination):
(3.1) ⊢*all(g, y)*→*some(g, y);*
(3.2) ⊢*no(g, y)*→*not all(g, y);*
(3.3) ⊢*all(g, y)*→*most(g, y);*
(3.4) ⊢*most(g, y)*→*some(g, y);*
(3.5) ⊢*at least half of the(g, y)*→*some(g, y);*
(3.6) ⊢*all(g, y)*→*at least half of the(g, y);*
(3.7) ⊢*at most half of the(g, y)*→*not all(g, y);*
(3.8) ⊢*fewer than half of the(g, y)*→*not all(g, y);*
(3.9) ⊢*no(g, y)*→*fewer than half of the(g, y).*

Fact 4 (symmetry):
(4.1) ⊢*some*(*g, y*)↔*some*(*y, g*);

(4.2) $\vdash no(g, y) \leftrightarrow no(y, g)$.

The above facts are elementary knowledge in generalized quantifier theory [9,10] and first-order logic [11], so their proofs are omitted.

The Validity and Reducibility of Generalized Syllogisms Based on *EMO-3*

It is necessary to prove the validity of the generalized syllogism *EMO-3* before discussing its reducible relationships between/among other syllogisms.

Theorem 1 *(EMO-3):* The generalized syllogism no(t, y)$\wedge$most(t, g)$\rightarrow$not all(g, y) is valid.

Proof: Suppose that no(t, y) and most(t, g) are true, then $T \cap Y = \varnothing$ and $T \cap G > 0.5T$ are true in terms of Definition D7 and D9, respectively. Thus it can be concluded that $G \not\subseteq Y$ is true. This can be proven by reductio ad absurdum. Assuming $G \not\subseteq Y$ is not true. That is to say, $G \subseteq Y$ is true. Because we have obtained $T \cap Y = \varnothing$, it follows that $T \cap G = \varnothing$, which conflicts with $T \cap G > 0.5T$. So $G \subseteq Y$ is not true. It means that $G \not\subseteq Y$ is true. Hence not all(g, y) is true in virtue of Definition D8, just as expected. If the validity of one syllogism can be deduced from that of another one, it is said that there is a reducible relationship between the two syllogisms. Taking (2.1) in Theorem 2 as an example, it illustrates that the validity of generalized syllogism *EMO-4* can be inferred from that of the syllogism *EMO-3*. In other words, there is a reducible relationship between syllogism *EMO-3* and *EMO-4*. On these grounds, one can obtain Theorem 2 as follows.

Theorem 2: At least the following 20 valid syllogisms can be inferred from *EMO-3:*

(2.1) $\vdash$*EMO-3*$\rightarrow$*EMO-4*
(2.2) $\vdash$*EMO-3*$\rightarrow$*AEH-2*
(2.3) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*AEH-4*
(2.4) $\vdash$*EMO-3*$\rightarrow$*AMI-1*
(2.5) $\vdash$*EMO-3*$\rightarrow$*AMI-1*$\rightarrow$*MAI-4*
(2.6) $\vdash$*EMO-3*$\rightarrow$*AMI-3*
(2.7) $\vdash$*EMO-3*$\rightarrow$*AMI-3*$\rightarrow$*MAI-3*
(2.8) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*
(2.9) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EAH-1*
(2.10) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EMO-1*
(2.11) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EMO-1*$\rightarrow$*EMO-2*
(2.12) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EAH-1*$\rightarrow$*AAS-1*
(2.13) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EAH-1*$\rightarrow$*AAS-1*$\rightarrow$*AFO-2*
(2.14) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EAH-1*$\rightarrow$*AAS-1*$\rightarrow$*FAO-3*
(2.15) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*AEO-2*
(2.16) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*AEH-4*$\rightarrow$*AEO-4*
(2.17) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EAO-2*
(2.18) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EAH-1*$\rightarrow$*EAO-1*
(2.19) $\vdash$*EMO-3*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EAH-1*$\rightarrow$*AAS-1*$\rightarrow$*AAI-1*
(2.20) $\vdash$*EMO-3*$\rightarrow$*AMI-1*$\rightarrow$MAI-4$\rightarrow$*AAI-4*

Proof:

[1] $\vdash no(t, y) \wedge most(t, g) \rightarrow not\ all(g, y)$ (i.e. *EMO-3*, Axiom A2)
[2] $\vdash no(y, t) \wedge most(t, g) \rightarrow not\ all(g, y)$ (i.e. *EMO-4*, by [1] and Fact (4.2))
[3] $\vdash \neg not\ all(g, y) \wedge no(t, y) \rightarrow \neg most(t, g)$ (by [1] and Rule 4)
[4] $\vdash all(g, y) \wedge no(t, y) \rightarrow at\ most\ half\ of\ the(t, g)$ (i.e. *AEH-2*, by [3], Fact (2.2) and (2.5))
[5] $\vdash all(g, y) \wedge no(y, t) \rightarrow at\ most\ half\ of\ the(t, g)$ (i.e. *AEH-4*, by [4] and Fact (4.2))
[6] $\vdash \neg not\ all(g, y) \wedge most(t, g) \rightarrow \neg no(t, y)$ (by [1] and Rule 5)
[7] $\vdash all(g, y) \wedge most(t, g) \rightarrow some(t, y)$ (i.e. *AMI-1*, by [6], Fact (2.2) and (2.3))
[8] $\vdash all(g, y) \wedge most(t, g) \rightarrow some(y, t)$ (i.e. *MAI-4*, by [7] and Fact (4.1))
[9] $\vdash all\neg(t, y) \wedge most(t, g) \rightarrow some\neg(g, y)$ (by [1], Fact (1.2) and (1.4))
[10] $\vdash all(t, D-y) \wedge most(t, g) \rightarrow some(g, D-y)$ (i.e. *AMI-3*, by [9] and D3)
[11] $\vdash all(t, D-y) \wedge most(t, g) \rightarrow some(D-y, g)$ (i.e. *MAI-3*, by [10] and Fact (4.1))
[12] $\vdash no\neg(g, y) \wedge all\neg(t, y) \rightarrow at\ most\ half\ of\ the(t, g)$ (by [4], Fact (1.1) and (1.2))
[13] $\vdash no(g, D-y) \wedge all(t, D-y) \rightarrow at\ most\ half\ of\ the(t, g)$ (i.e. *EAH-2*, by [12] and D3)
[14] $\vdash no(D-y, g) \wedge all(t, D-y) \rightarrow at\ most\ half\ of\ the(t, g)$ (i.e. *EAH-1*, by [13] and Fact (4.2))
[15] $\vdash \neg at\ most\ half\ of\ the(t, g) \wedge no(g, D-y) \rightarrow \neg all(t, D-y)$ (by [13] and Rule 4)
[16] $\vdash most(t, g) \wedge no(g, D-y) \rightarrow not\ all(t, D-y)$ (i.e. *EMO-1*, by [15], Fact (2.1) and (2.6))
[17] $\vdash most(t, g) \wedge no(D-y, g) \rightarrow not\ all(t, D-y)$ (i.e. *EMO-2*, by [16] and Fact (4.2))
[18] $\vdash all\neg(D-y, g) \wedge all(t, D-y) \rightarrow at\ least\ half\ of\ the\neg(t, g)$ (by [14], Fact (1.2) and (1.8))
[19] $\vdash all(D-y, D-g) \wedge all(t, D-y) \rightarrow at\ least\ half\ of\ the(t, D-g)$ (i.e. *AAS-1*, by [18] and D3)
[20] $\vdash \neg at\ least\ half\ of\ the(t, D-g) \wedge all(D-y, D-g) \rightarrow \neg all(t, D-y)$ (by [19] and Rule 4)
[21] $\vdash fewer\ than\ half\ of\ the(t, D-g) \wedge all(D-y, D-g) \rightarrow not\ all(t, D-y)$ (i.e. *AFO-2*, by [20], Fact (2.1) and (2.8))
[22] $\vdash \neg at\ least\ half\ of\ the(t, D-g) \wedge all(t, D-y) \rightarrow \neg all(D-y, D-g)$ (by [19] and Rule 5)
[23] $\vdash fewer\ than\ half\ of\ the(t, D-g) \wedge all(t, D-y) \rightarrow not\ all(D-y, D-g)$ (i.e. *FAO-3*, by [22], Fact (2.1) and (2.8))
[24] $\vdash at\ most\ half\ of\ the(t, g) \rightarrow not\ all(t, g)$ (by Fact (3.7))
[25] $\vdash all(g, y) \wedge no(t, y) \rightarrow not\ all(t, g)$ (i.e. *AEO-2*, by [4], [24] and Rule 1)
[26] $\vdash all(g, y) \wedge no(y, t) \rightarrow not\ all(t, g)$ (i.e. *AEO-4*, by [5], [24]

and Rule 1)
[27] $\vdash no(g, D-y)\wedge all(t, D-y)\rightarrow not\ all(t, g)$ (i.e. *EAO-2*, by [13], [24] and Rule 1)
[28] $\vdash no(D-y, g)\wedge all(t, D-y)\rightarrow not\ all(t, g)$ (i.e. *EAO-1*, by [14], [24] and Rule 1)
[29] $\vdash at\ least\ half\ of\ the(t, D-g)\rightarrow some(t, D-g)$ (by Fact (3.5))
[30] $\vdash all(D-y, D-g)\wedge all(t, D-y)\rightarrow some(t, D-g)$ (i.e. *AAI-1*, by [19], [29] and Rule 1)
[31] $\vdash all(t, g)\rightarrow most(t, g)$ (by Fact (3.3))
[32] $\vdash all(g, y)\wedge all(t, g)\rightarrow some(y, t)$ (i.e. *AAI-4*, by [8], [31] and Rule 3)

It can be seen that the above 14 valid generalized syllogisms be deduced in line with the validity of the generalized syllogism *EMO-3* from Step 1 to Step 23. Due to the fact that Aristotelian syllogisms are special cases of generalized syllogisms [12], the above 6 valid Aristotelian syllogisms also can be derived from Step 24 to Step 32 in terms of subsequent weakening rule and antecedent strengthening rule. So far, a total of 20 valid syllogisms have been obtained from the syllogism *EMO-3*. If one similarly continues the above reductive operations, the other valid syllogisms can be deduced from the syllogism *EMO-3*.

Conclusion and Future Work

To explore the reducibility of non-trivial generalized syllogisms with the quantifiers in Square{not all} and Square{most}, this paper first gives the formalization of generalized syllogisms on the basis of set theory, and then proves the validity of the generalized syllogism *EMO-3* by first-order logic and generalized quantifier theory; Finally, with the help of some reductive operations, the other 20 valid generalized syllogisms are deduced from the syllogism *EMO-3*. In other words, there are the reducible relationships between/among the above 21 valid syllogisms. The reason for this is that any quantifier in a square can define the other three quantifiers. This research contributes to breaking through the existing research paradigms in linguistics and logic, integrating the latest research methods and findings, and providing macro-level ideas and concrete approaches for constructing a truly automated natural language processing system, as well as enhancing interpersonal dialogue and human-computer interaction. Any quantifier and its three negation quantifiers can form a modern square. Can the research method in this paper be applied to study other generalized syllogisms with quantifiers in other squares? These questions are meaningful and worthy of further exploration.

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